

# Slip and boundary conditions on MHD nanofluid flow of saturated porous media over a radiating stretching sheet with heat source/sink and chemical reaction parameter

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## Abstract:

Slip and boundary conditions on MHD nanofluid flow of saturated porous media over a radiating stretching sheet with heat source/sink and Chemical reaction parameter is presented. The similarity solution is used to transform the problem under consideration into a boundary value problem of coupled ordinary differential equations, which are solved numerically by using the finite difference method. Numerical computations are carried out for the non-dimensional physical parameter. The results are analyzed for the effect of different physical parameters such as chemical reaction parameter, magnetic field parameter, Prandtl number, thermophoresis parameter, Brownian motion parameter, convection-radiation parameter, Lewis number, hydrodynamic (momentum) slip parameter, convection-diffusion parameter, convection-conduction parameter, on the dimensionless velocity, temperature and nanoparticle concentration fields and are presented through graphs.

**Key words:** Chemical reaction parameter, MHD, chemical reaction parameter, heat source/sink, hydrodynamic (momentum) slip parameter.

## Introduction

Nanofluid have their major applications in heat transfer including micro electronics, fluids pharmaceutical processes, domestic refrigerators, chiller, nuclear reactor coolant, space technology and in boiler flue, gases temperature reduction. They demonstrate enhanced thermal conductivity and convective heat transfer coefficient, counter balance to the base fluid. Many research papers have been published on nanofluids to understand their performance and due to their diverse applications in science and engineering. A comprehensive study of convective transport in nanofluids was made by Buongiorno and Hu [1] and Buongiorno [2]. Kuznetsov and Nield [3] presented a similarity solution of natural convective boundary layer flow of a nano fluid past a vertical plate. Fluid flow and heat transfer through porous medium have been of considerable interest, especially in the past decade. This is primarily because of numerous applications of flow through porous media such as storage of radioactive nuclear waste materials transfer, separation processes in chemical industries, filtration, thermal insulation engineering, packed- bed reactors, transpiration cooling, transport processes in aquifers, ground water pollution, etc. This topic is of vital importance in all these applications, thereby generating the need for a full understanding of transport processes through porous media. Srinivas Maripala, Kishan.N[4], studied the “unsteady MHD flow and heat transfer of nanofluid over a permeable shrinking sheet with thermal radiation and chemical reaction” However, such studies for nanofluid are rarely available in the literature especially when conjugate effects of heat and mass transfer are considered [5].

Gorla and Chamkha [6] studied natural convection flow past a horizontal plate in a porous medium filled with a nanofluid. Balla and Kishan [7] studied the unsteady natural convection flow of a nanofluid past a vertical cone with thermal radiation under the influence of magnetic fluid. The study of hydrodynamic flow and heat transfer over a stretching sheet may find its application to sheet extrusion in order to make flat plastic sheets. In doing so, it is important to investigate cooling and heat transfer for the improvement of the final products. The conventional fluids such as water and air are among the most widely used fluids as the cooling medium. However, the rate of heat exchange achievable by the above fluids is realized to be unsuitable for certain sheet materials. Thus, in recent years, it has been proposed to alter flow kinematics that it leads to a slower rate of solidification as compared with water. Among the techniques to control flow kinematics, the idea of using magnetic field appears to be the most attractive one both because of its ease of implementation and also because of its intrusive nature. The fluid mechanics properties desired for an outcome of such a process would mainly depend on two aspects, one is the rate of cooling of liquid used and the other is the rate of stretching. The rate of cooling and the desired properties of the end product can be controlled by the use of electrically conducting fluid and applications of magnetic fields. Dessie and Kishan [8] studied the heat transfer over stretching sheet embedded in porous medium with variable viscosity, viscous dissipation and heat source/sink. Recently, Srinivas Maripala and Kishan Naikoti [23] studied the, MHD convection slip flow of a thermosolutal nanofluid in a saturated porous media over a radiating stretching sheet with heat source/sink when chemical reaction

parameter is absent. In the present investigation is to study the chemical reaction effects of magnetohydrodynamics convection slip flow of a thermosolutal nanofluid in a saturated porous media over a radiating stretching sheet with heat source/sink. The governing equations are solved by using the implicit finite difference scheme using along with the Gauss-Siedel iteration method. The C-programming code is used to compute the values for the same.

### Governing Nanofluid Transport Model

Consider a two-dimensional regime with a coordinate system with the  $\bar{x}$ -axis aligned horizontally and the  $\bar{y}$ -axis is normal to it. A transverse magnetic field  $B_0$  acts normal to the bounding surface. The magnetic Reynolds number is small so that the induced magnetic field is effectively negligible when compared to the applied magnetic field. We neglect the electric field associated with the polarization of charges and Hall effects. It is further assumed that the left of the plate is heated by the convection from the hot fluid of temperature  $T_f (> T_w > T_\infty)$  which provides a variable heat transfer coefficient  $h_f(\bar{x})$ . Consequently, a thermal convective boundary condition arises. It is further assumed that the concentration in the left of the plate  $C_f$  is higher than that of the plate concentration and free stream concentration  $C_\infty$  which provides a variable mass transfer coefficient  $h_m(\bar{x})$ . As a result, a mass convective boundary condition arises. The Oberbeck- Boussinesq approximation is utilized and the four field equations are the conservation of mass, momentum, thermal energy and the nanoparticles volume fraction. These equations can be written in terms of dimensional forms, extending the formulations of Buongiorno [15] and Makinde and Aziz [16]:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{1}$$

$$\rho_f (\bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}}) = \mu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\mu}{Kp} \bar{u} - \sigma B_0^2 \bar{u}, \tag{2}$$

$$\bar{u} \frac{\partial T}{\partial \bar{x}} + \bar{v} \frac{\partial T}{\partial \bar{y}} = \alpha \frac{\partial^2 T}{\partial \bar{y}^2} + \tau \left\{ D_B \frac{\partial C}{\partial \bar{y}} \frac{\partial T}{\partial \bar{y}} + \left( \frac{D_T}{T_\infty} \right) \left( \frac{\partial T}{\partial \bar{y}} \right)^2 \right\} - \frac{1}{\rho_f c_f} \frac{\partial q_r}{\partial \bar{y}} + \frac{Q_0}{(\rho c)_f} (T - T_\infty) \tag{3}$$

$$\bar{u} \frac{\partial C}{\partial \bar{x}} + \bar{v} \frac{\partial C}{\partial \bar{y}} = D_B \frac{\partial^2 C}{\partial \bar{y}^2} + \left( \frac{D_T}{T_\infty} \right) \frac{\partial^2 T}{\partial \bar{y}^2} - k_c (C - C_\infty) \tag{4}$$

The appropriate boundary conditions are following Datta[27] and Karniadakis et al.[28].

$$\begin{aligned} \bar{u} = \bar{u}_w + \bar{u}_{slip}; \bar{v} = 0, -k \frac{\partial T}{\partial \bar{y}} = h_f (T_f - T), -D_B \frac{\partial C}{\partial \bar{y}} = h_m (C_f - C) \text{ at } \bar{y} = 0, \\ \bar{u} \text{ tends to } 0, T \text{ tends to } T_\infty, C \text{ tends to } C_\infty \text{ as } \bar{y} \text{ tend to } \infty \end{aligned} \tag{5}$$

Here  $\alpha = k / (\rho c)_f$  thermal diffusivity of the fluid;  $\tau = (\rho c)_p / (\rho c)_f$ , ratio of heat capacity of the nanoparticle and fluid;  $Kp$ : permeability of the medium;  $(\bar{u}, \bar{v})$ : velocity components along  $\bar{x}$  and  $\bar{y}$  axes,  $\bar{u}_w = U_r (\bar{x}/L)$  velocity of the plate,  $L$ : characteristic length of the plate,  $\bar{u}_{slip} = N_1 v \frac{\partial \bar{u}}{\partial \bar{y}}$ : linear slip velocity,  $N_1$  velocity slip factor with dimension s/m,  $\rho_f$ : density of the base fluid,  $\sigma$ : electric conductivity,  $\mu$ : dynamic viscosity of the base fluid,  $\rho_p$ : density of the nanoparticles,  $(\rho C p)_f$ : effective heat capacity of the fluid,  $(\rho C p)_p$  effective heat capacity of the nanoparticle material,  $Q_0$  volumetric rate of heat generation/absorption,  $\epsilon$ : porosity,  $D_B$ : Brownian diffusion coefficient,  $D_T$ : thermophoretic diffusion coefficient and  $q_r$ : radiative heat transfer in  $\bar{y}$ -direction. We consider the fluid to be a gray, absorbing-emitting but non scattering medium. We also assume that the boundary layer is optically thick and the Rosseland approximation or diffusion approximation for radiation is valid [19, 20]. Thus, the radioactive heat flux for an optically thick boundary layer (with intensive absorption), as elaborated by Sparrow and Cess [21], is defined as  $q_r = \left( \frac{-4\sigma_1}{3k_1} \right) \left( \partial T^4 / \partial \bar{y} \right)$  where  $\sigma_1 (= 5.67 \times 10^{-8} W / m^2 K^4)$  is the Stefan-Boltzmann constant and  $k_1 (m^{-1})$  Rosseland mean absorption coefficient. Purely analytical solutions to the partial differential boundary value problem defined by (2)–(4) are not possible. Even a numerical solution is challenging. Hence we aim to transform the problem to a system of ordinary differential equations. We define the following dimensionless transformation variables:

$$\eta = \frac{\bar{y}}{\sqrt{Kp}}, \quad \psi = U_r (\bar{x}/L) \sqrt{Kp} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, \quad \varphi(\eta) = \frac{C - C_\infty}{C_f - C_\infty}, \tag{6}$$

where  $L$  is the characteristic length. From (4), we have  $T = T_\infty \{1 + (T_r - 1)\theta\}$ , where  $T_r = T_f / T_\infty$  (the wall temperature excess ratio parameter) and hence  $T^4 = T_\infty^4 \{1 + (T_r - 1)\theta\}^4$ . Substitution of (6) into (2)–(4) generates the following similarity equations:

$$f''' + Re Da (ff'' - f'^2 - M f') - f' = 0 \tag{7}$$

$$\theta'' + Re Pr Da f \theta' + Pr [Nb \theta' f \varphi' + Nt \theta'^2] + \frac{4}{3R} [1 + (Tr - 1)\theta]^3 \theta' + Q\theta = 0 \tag{8}$$

$$\varphi'' + Re Le Da f \varphi' + \frac{Nt}{Nb} \theta'' - \gamma \varphi = 0 \tag{9}$$

The relevant boundary conditions are

$$\begin{aligned} f(0) = 0, f'(0) = 1 + af''(0), & \quad \theta'(0) = -Nc[1 - \theta(0)], \\ \varphi'(0) = -Nd[1 - \varphi(0)], & \quad f'(\infty) = \theta(\infty) = \varphi(\infty) = 0, \end{aligned} \tag{10}$$

where primes denote differentiation with respect to  $\eta$ . The thermo physical dimensionless parameters arising in (7)–(9) are defined as follows:  $Re = U_r L / \nu$  is the Reynolds number,  $Da = k_p / L^2$  is the Darcy number,  $M = \sigma B_0^2 L / U_r \rho$  is the magnetic field parameter,  $Pr = \nu / \alpha$  is the Prandtl number,  $Nt = \tau D_T (T_f - T_\infty) / \nu T_\infty$  is the thermophoresis parameter,  $Nb = \tau D_B (C_f - C_\infty) / \nu$  is the Brownian motion parameter,  $R = kk_1 / 4\sigma_1 T_\infty^3$  is the convection-radiation parameter,  $Le = \nu / D_B$  is the Lewis number,  $a = N_1 \nu / \sqrt{K_p}$  is the hydrodynamic (momentum) slip parameter,  $Nd = h_m \sqrt{K_p} / D_B$  is the convection-diffusion parameter,  $Nc = h_f \sqrt{K_p} / k$  is the convection-conduction parameter and the chemical reaction parameter  $\gamma = k_c (C - C_\infty)$ . Quantities of physical interest are the local friction factor,  $C_{f_x}$ , the local Nusselt number,  $Nu_x$  and the local Sherwood number,  $Sh_x$ . Physically,  $C_{f_x}$  represents the wall shear stress,  $Nu_x$  defines the heat transfer rates and  $sh_x$  defines the mass transfer rates:

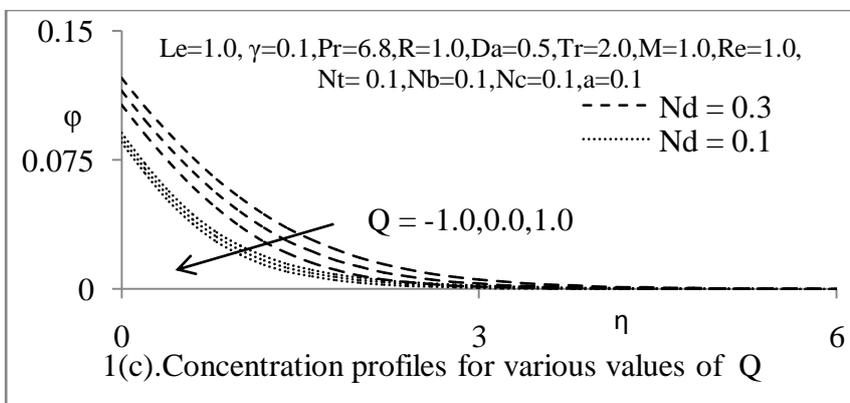
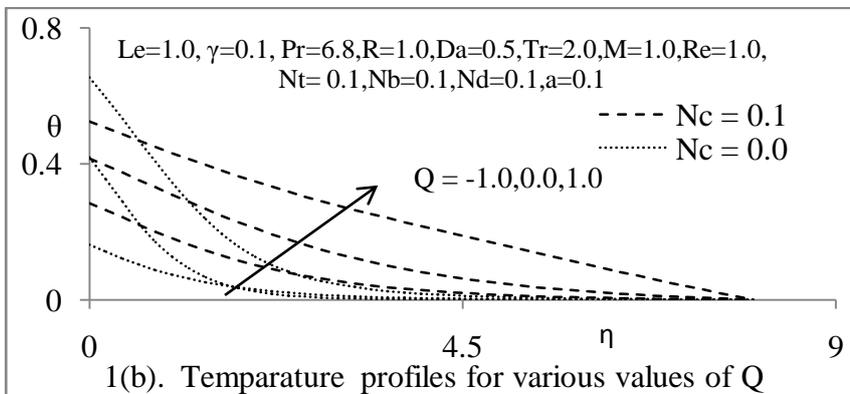
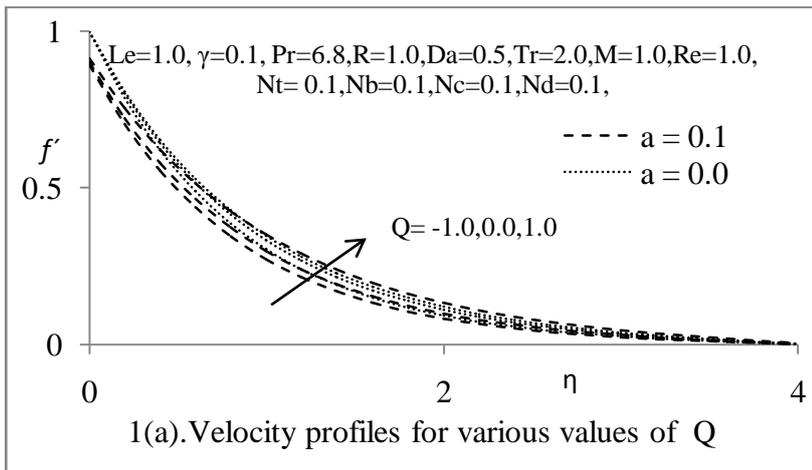
$$\begin{aligned} C_{f_x} Re_x^{-1} Da_x^{0.5} &= 2f''(0), \quad Sh_x Da_x^{0.5} = -\varphi'(0) \\ Nu_x Da_x^{0.5} &= -[1 + \frac{4}{3R} \{1 + (Tr - 1)\theta(0)\}^3] \theta'(0) \end{aligned} \tag{11}$$

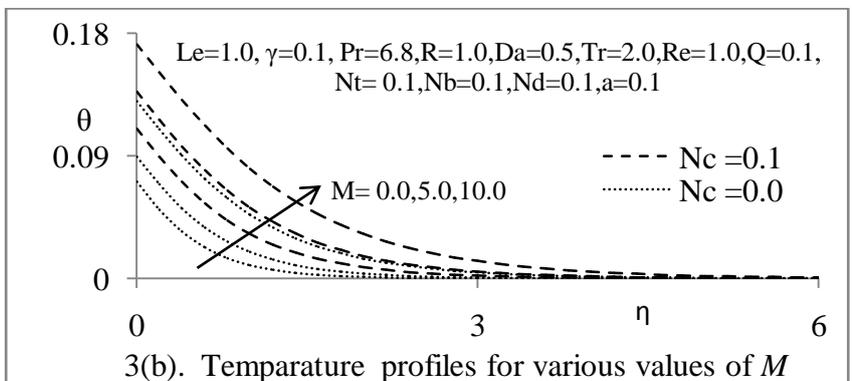
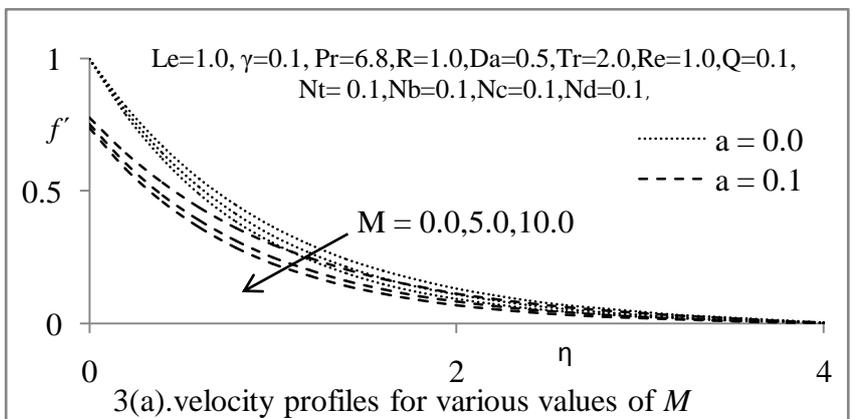
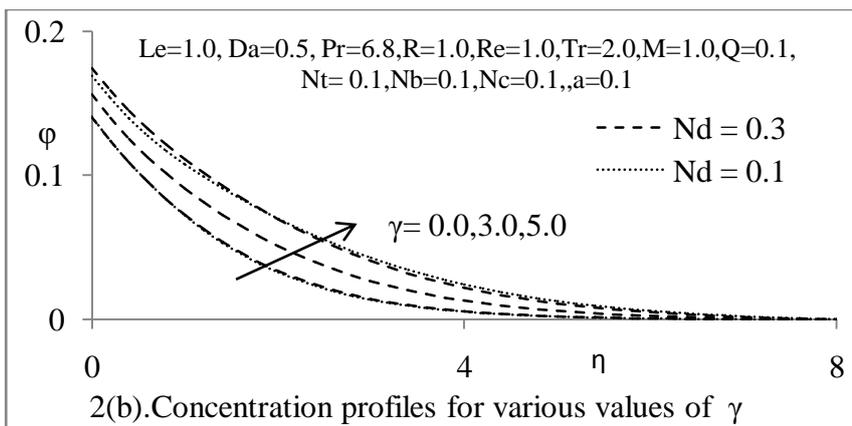
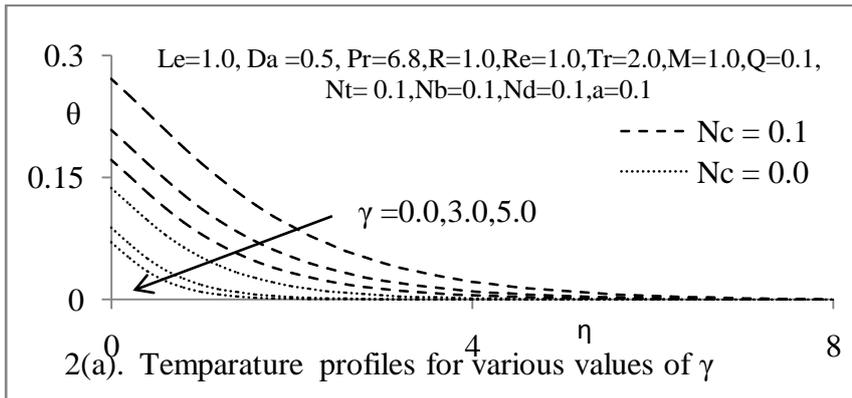
### Results and discussion

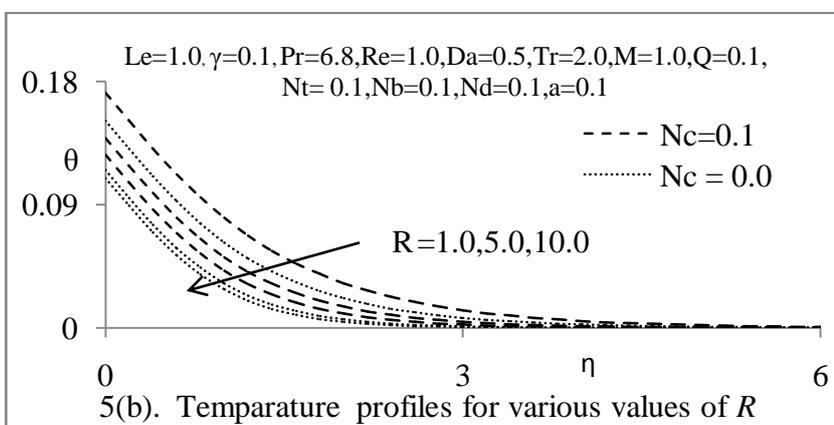
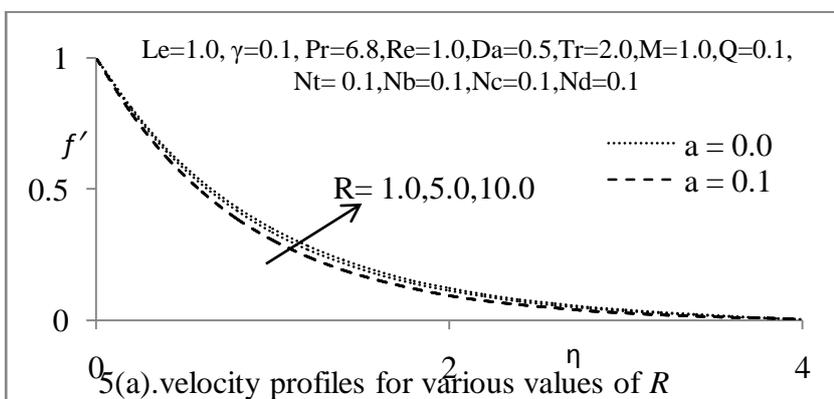
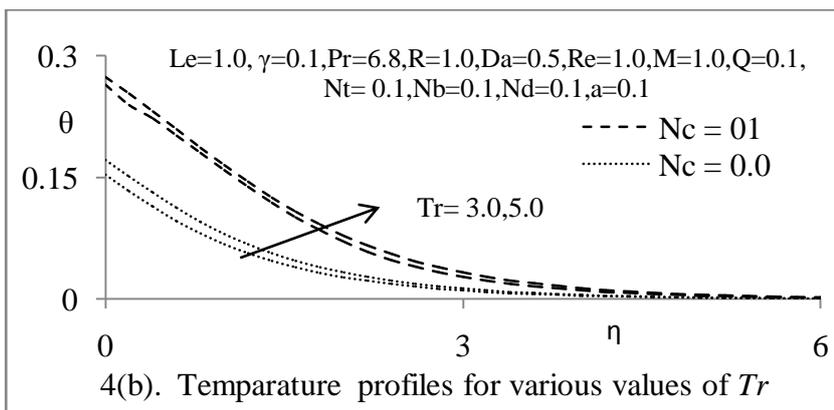
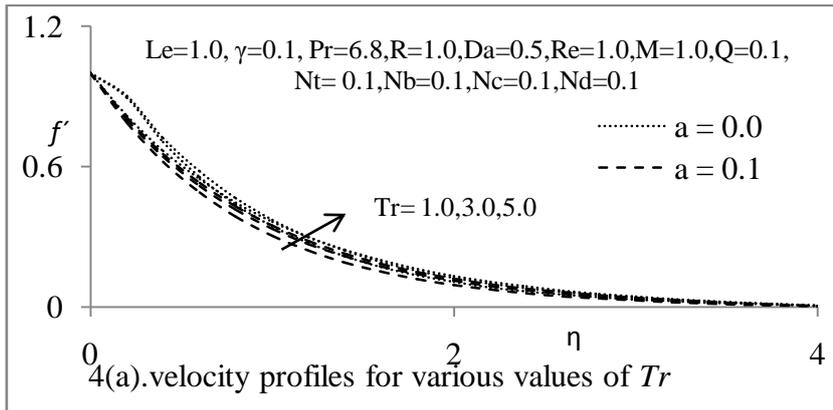
The numerical simulation of equations (7)-(9) subject to the boundary condition (10) are carried out for various values of the physical parameters such as Reynolds number  $Re$ , magnetic field parameter  $M$ , Prandtl number  $Pr$ , thermophoresis parameter  $Nt$ , Brownian motion parameter  $Nb$ , convection-radiation parameter  $R$ , Lewis number  $Le$ , hydrodynamic (momentum) slip parameter  $a$ , convection-diffusion parameter  $Nd$ , convection-conduction parameter  $Nc$ . In order to get a physical understanding of the problem the results have been performed for the velocity, temperature and concentration profiles. The results are presented graphically in figures 1-10

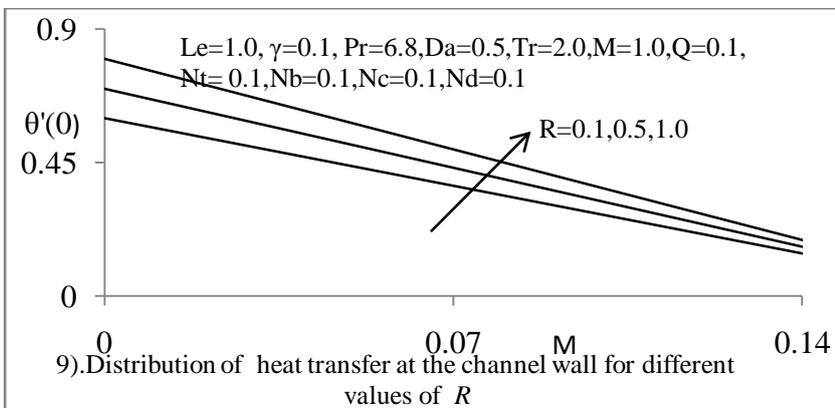
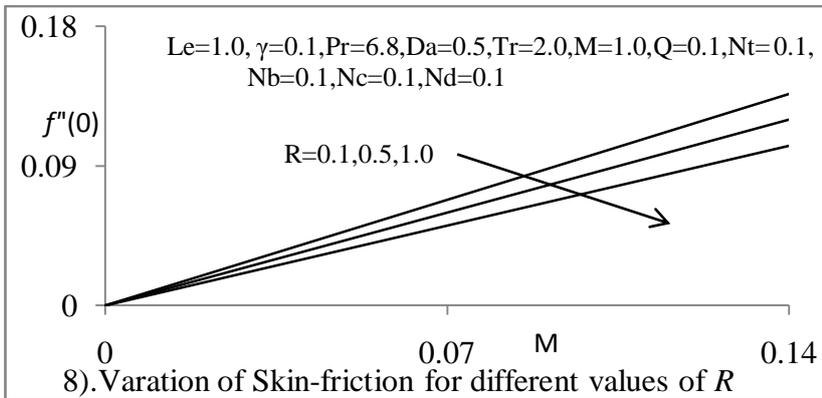
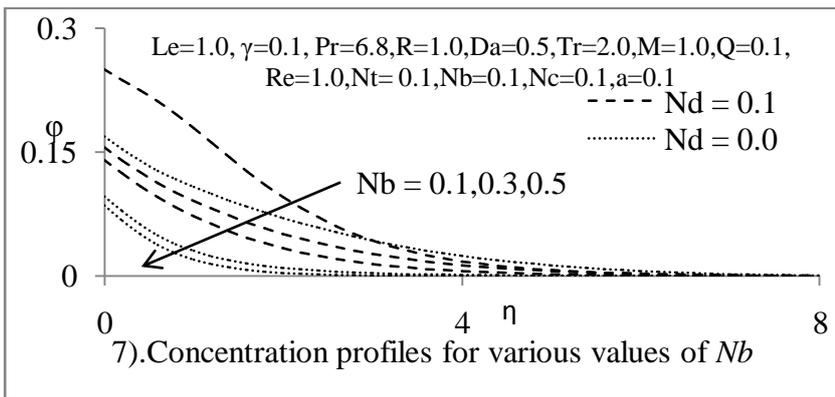
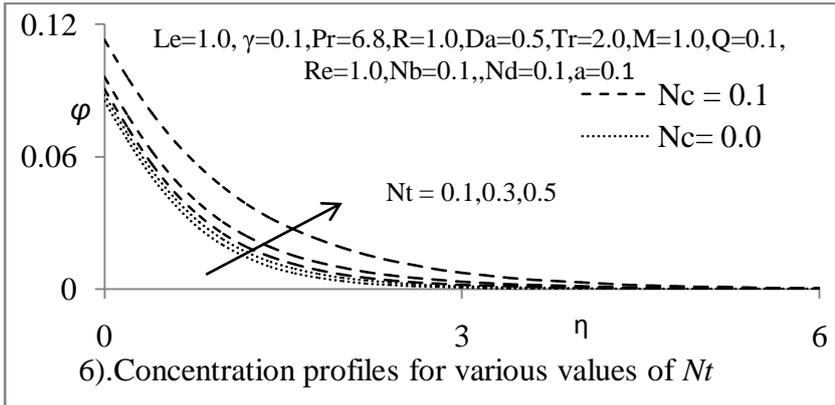
Figures 1(a) to 1(c) illustrate the velocity, temperature and concentration profiles for different values of heat source/sink parameter  $Q$ . From figure 1(a), it reveal that with the effect of heat source/sink parameter ( $Q < 0$ ), the velocity profiles decreases and the velocity profiles increase with heat source ( $Q > 0$ ). There is a significant variation is observed from figure 1(b) the temperature profiles decreases with heat source/sink parameter ( $Q < 0$ ) on the contrary, the temperature profiles increases with heat source/sink ( $Q > 0$ ) for both cases with and without convection-conduction parameter  $Nc$ . Figure 1(c) represents the effect of heat source/sink parameter  $Q$ , for different values of convection diffusion parameter  $Nd$  on concentration profiles. The concentration profiles increase in case of heat source/sink  $Q < 0$ , while the concentration profiles decreases with heat source/sink parameter  $Q > 0$ . It is interested to note that the heat sources effect is higher for higher conduction-diffusion parameter  $Nd$ . Therefore, the thermal and concentration boundary layer thickness are enhanced. Figure 3 depicts the influence of chemical reaction on the dimensionless temperature and solutal concentration profiles with the fixed values of other parameters. It is obvious that an increase in the chemical reaction parameter results a decreasing in the solutal concentration profile. The distribution of solutal concentration becomes weak in the presence of chemical reaction. So, the solutal concentration boundary layer becomes thin as the chemical reaction parameter increases. From Fig. 2, it is observed that the chemical reaction influences the solutal concentration field. However, it has a minor effect on thermal diffusion. This explains the minor influence of chemical reaction on temperature profile. It is worth mentioning here that, the large values of  $\gamma$  shows small changes on temperature field. The velocity, temperature and concentration display in 3(a) and 3(b). It is depicted that the effect of magnetic fluid is to decrease the velocity field for the cases slip and no slip boundary conditions. It is due to Lorentz forces in the saturated porous medium. When the strength of the magnetic field increase the velocity gradient at the interface has been diminishes, the temperature profiles increases with the increase of magnetic field  $M$ . Figure 4(a) and 4(b) demonstrates that an increase wall temperature parameter  $Tr$  increases velocity as well as temperature profiles. It can be also noticed that the convection parameter  $Nc$  increases the temperature profiles. Figures 5(a) and 5(b) depict the influence of thermal radiation parameter  $R$  on velocity and temperature profiles. It can be seen from the figure that the velocity profiles increase with the increase of Reynolds number  $Re$ , while the temperature profiles with the increase of  $R$ . This is due to the Radiation term appeared in equation (8) is inversely proportional to the radiation parameter  $R$ . Thus, small  $R$  signifies a large radiation effect while  $R$  tends to infinite correspond to no radiation effect. Evidently the presence of thermal radiation flux is demonstrated to heat the thermal boundary layer significantly and is benefited to material characteristic. Figure 6, represents the effect of thermophorasis parameter  $Nt$  when conduction parameter

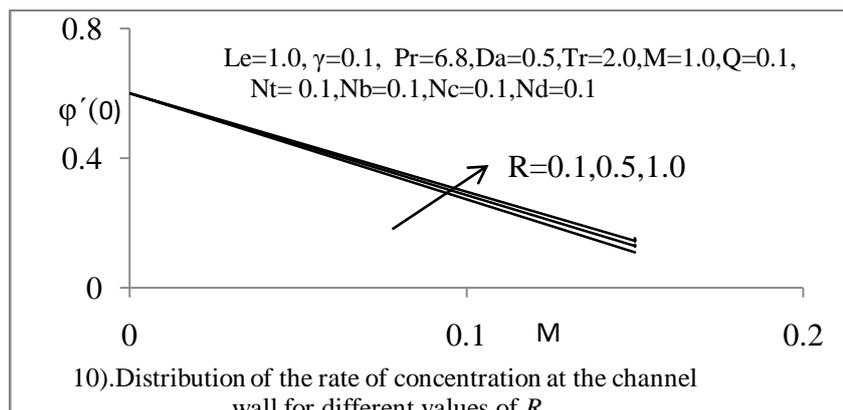
$Nc=0$ (without biot number),  $Nc= 0.1$ (with biot number) with influence of thermophoresis parameter  $Nt$ . The concentration profiles increase with increase of thermophoresis parameter  $Nt$ . It is worth mentioning that the thermophoresis parameter  $Nt$  effect is more in the presence of Biot number  $Nc$ . The effect of Brownian motion parameter  $Nb$  is to display on concentration profiles in Figure 7. It can be seen that the Brownian motion parameter  $Nb$  effects lead to decrease the concentration profiles. It is evident from Figure.8, the variation of skin friction parameter  $f''(0)$  decreases with the increase of radiation parameter  $R$  and increases with the increase of magnetic parameter  $M$ . The Figure 9 is done for the values of  $\theta'(0)$  verses the magnetic field parameter  $M$ . From figure it revealed that the heat transfer coefficient  $\theta'(0)$  is to increase with the increase of  $R$  and reduces the values with increase of values of magnetic parameter  $M$ . Figure 10 is plotted for the Sherwood number  $\phi'(0)$  verses the magnetic parameter  $M$  for different values of radiation parameter  $R$ . The results indicate that an increasing magnetic parameter  $M$  is to decrease the Sherwood parameter coefficients  $\phi'(0)$  values. It is also noticed that the Sherwood parameter coefficients  $\phi'(0)$  values increasing with the increase of radiation parameter  $R$ .











### Conclusion

In the present investigation, an analysis in order to study the chemical reaction effects of magnetohydrodynamics convection slip flow of a thermosolutal nanofluid in a saturated porous media over a radiating stretching sheet with heat source/sink. In this study the density is dependent on velocity, temperature and concentration profiles. The governing equations are transformed to the high non-linear ordinary differential equations by the use of similarity transformation and are solved analytically using finite difference method along with the Gauss-seidel method. The physical parameters such as Reynolds number, chemical reaction parameter, magnetic field parameter, Prandtl number, thermophoresis parameter, Brownian motion parameter, convection-radiation parameter, Lewis number, hydrodynamic (momentum) slip parameter, convection-diffusion parameter, convection-conduction parameter on velocity, temperature and concentration profiles are depicted and discussed in this paper.

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