

Determinations of A Square Fuzzy Soft Matrices

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Abstract

In this paper we introduce fuzzy determinant ordering with fuzzy soft matrices using the structure of M_n (FS) the set of $(n \times n)$ fuzzy determinant ordering with fuzzy soft matrices is introduced and also we introduce the concept of fuzzy determinant partitions of M_n (FS), properties of fuzzy determinant ordering.

Keywords : Soft set, fuzzy soft set (FSS), fuzzy soft Matrix (FSM), determinant of a square fuzzy soft matrix.

I. Introduction

The concept of fuzzy set was introduced by Zadeh [8] in 1965. Jian Miao Chen [2] introduced the fuzzy matrix partial ordering and generalized inverse. Bertoluzza [1] introduced the distributivity of t – norm and t – conorms. In 1995, Ragab. M. Z and Emam E.G. [7] introduced the determinant and adjoint of a square fuzzy matrix. Meenakshi A.R and Kokilavani R. [3] introduced the concept of fuzzy 2 normed linear spaces. Nagoorani A and Kalyani G [5] introduced the fuzzy matrix m – ordering. Zhou Min na [9] introduced the characterizations of the minus ordering in fuzzy matrix set. In this paper, we introduce the concept of fuzzy determinant ordering with fuzzy soft matrices. The purpose of the introduction is to explain determinant ordering with fuzzy soft matrices and partitions of M_n (FS). In section 4, properties of determinant ordering with fuzzy soft matrices.

II. Preliminaries

In this section, we recall some basic essential notion of fuzzy soft set theory and fuzzy soft matrices. 2.1 soft set [13]. Let U be an initial set and E be a set of parameters. Let $P(U)$ denotes the power set of U . Let $A \subseteq E$. A pair (F_A, E) is called a soft set over U , where F_A is a mapping given by $F_A: E \rightarrow P(U)$. such that $F_A(e) = \emptyset$ if $e \notin A$. Here F_A is called approximate fraction of the soft set (F_A, E) . The set $F_A(e)$ is called e – approximate value set which consist of related objects of the parameter $e \in E$. In other words, a soft set over U is a parameterized family of subsets of the universe U .

Example 2.1

Let $U = \{u_1, u_2, u_3, u_4\}$ be a set of four shirts and $E = \{\text{blue } (e_1), \text{ green } (e_2), \text{ yellow } (e_3)\}$ be a set of parameters if $A = \{e_1, e_3\} \subseteq E$. Let $F_A(e_1) = \{u_1, u_3, u_4\}$ and $F_A(e_3) = \{u_2, u_2, u_4\}$ then we write the soft set $(F_A, E) = \{(e_1, \{u_1, u_3, u_4\}), (e_3, \{u_1, u_2, u_4\})\}$ over U which describe the “Colour of the shirts” which Mr. X is going to buy.

We may represent the soft set in the following form.

U	Blue(e1)	Green(e2)	Yellow(e3)
U ₁	1	0	1
U ₂	0	0	1
U ₃	1	0	0
U ₄	1	0	1

Table : 2.1.1

2.2 FUZZY SOFT SET [12]

Let U be an initial universe set and E be a set of parameters. Let P(U) denotes the set of all fuzzy sets of U. Let ACE. A pair (F_A,E) is called a fuzzy soft set (FSS) over U, where F_A is a mapping given by F_A : E → P(U) such that F_A e = Φ if e ∈ E, Φ is a null fuzzy set.

Example 2.2

Consider the example 2.1 here we cannot express with only two real numbers 0 and 1, we can characterized it by a memberships function instead of crisp number 0 and 1, which associate with each element a real number in the interval [0,1] then.

$$(F_A, E) = \{ F_A(e_1) = \{(u_1, 0.3), (u_2, 0.9), (u_3, 0.4), (u_4, 0.7)\},$$

$F_A(e_3) = \{(u_1, 0.5), (u_2, 0.6), (u_4, 0.8)\}$ is the fuzzy soft set representing the “Colour of the Shirts” which r-X is going to buy. We may represent the fuzzy soft set in the following form.

U	Blue(e1)	Green(e2)	Yellow(e3)
U ₁	0.3	0.0	0.5
U ₂	0.9	0.0	0.6
U ₃	0.4	0.0	0.0
U ₄	0.7	0.0	0.8

Table 2.2

2.3. The complement of a fuzzy soft set [1]

The complement of a fuzzy soft set (F,A) is denoted by $(F,A)^c$ and is defined by

$$(F,A)^c = (F^c,A) \text{ where, } F^c : A \rightarrow P(U) \text{ is mapping given by } F^c(e) = [F(e)]^c \quad \forall e \in A.$$

2.4 Fuzzy Soft Matrices (FSM) (11)

Let (F_A,E) be a fuzzy soft set over U , then a subset of $U \times E$ is uniquely defined by $R_A = \{(u,e)\}; e \in A, u \in F_A(e)$ which is called relation form of (F_A,E) the characteristic function of R_A is written by $\mu_{RA} : U \times E \rightarrow [0,1]$, where $\mu_{RA}(u,e) \in [0,1]$ is the membership value of $u \in E$ for each $e \in U$.

If $[\mu_{ij}] = \mu_{RA}(u_i, e_j)$, we can define a matrix

$$[\mu_{ij}]_{m \times n} = \begin{bmatrix} \mu_{11} & \mu_{12} & \dots & \mu_{1n} \\ \mu_{21} & \mu_{22} & \dots & \mu_{2n} \\ \dots & \dots & \dots & \dots \\ \mu_{m1} & \mu_{m2} & \dots & \mu_{mn} \end{bmatrix}$$

which is called an $m \times n$ soft matrix of the soft set (F_A,E) over U .

therefore we can say that a fuzzy soft set (F_A,E) is coniquely characterized by the matrix $[\mu_{ij}]_{m \times n}$ and both concepts are interchangeable.

The set $m \times n$ fuzzy soft matrices over U will be denoted by $FSM_{m \times n}$.

Example 2.3

Assume that $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a universal set and $E = \{e_1, e_2, e_3, e_4\}$ is a set of an parameters.

$$F(e_1) = \{(u_1, 0.3), (u_2, 0.4), (u_3, 0.6), (u_4, 0.7), (u_5, 0.5)\}$$

$$F(e_2) = \{(u_1, 0.2), (u_2, 0.7), (u_3, 0.1), (u_4, 0.8), (u_5, 0.6)\}$$

$$F(e_3) = \{(u_1, 0.1), (u_2, 0.3), (u_3, 0.5), (u_4, 0.4), (u_5, 0.9)\}$$

Then the fuzzy soft matrix $[\mu_{ij}]$ can be written as

$$[\mu_{ij}]_{m \times n} = \begin{bmatrix} 0.3 & 0.2 & 0.0 & 0.1 \\ 0.4 & 0.7 & 0.0 & 0.3 \\ 0.6 & 0.1 & 0.0 & 0.5 \\ 0.7 & 0.8 & 0.0 & 0.4 \\ 0.5 & 0.6 & 0.0 & 0.9 \end{bmatrix}$$

2.4 Definition 2.4 [15]

Let $U = \{c_1, c_2, \dots, c_m\}$ be the universal set and E be the set of parameters given by $E = \{e_1, e_2, e_3, \dots, e_n\}$. let the set of all $m \times n$ fuzzy soft matrices over U be $FSM_{m \times n}$. Let $A, B \in FSM_{m \times n}$. Where $A = [a_{ij}]_{m \times n}$, $a_{ij} = (\mu_{j1}c(i), \mu_{j2}c(i))$ and $B = [b_{ij}]_{m \times n}$, $b_{ij} = (\lambda_{j1}c(i), \lambda_{j2}c(i))$. To avoid degenerate case we assume that $\min(\mu_{j1}c(i), \lambda_{j1}c(i)) \geq \max(\mu_{j2}c(i), \lambda_{j2}c(i))$ for all i and j . we define the operation "addition(+)" between, $A+B=C$, where $C = [c_{ij}]_{m \times n}$, $c_{ij} = (\max(\mu_{j1}c(i), \lambda_{j1}c(i)), \min(\mu_{j2}c(i), \lambda_{j2}c(i)))$. If $\mu_{j2}c(i) = \lambda_{j2}c(i) = 0 \quad \forall i, j$, then one definition reduce to $A+B=C$, where $C = [c_{ij}]_{m \times n}$.

$$C_{ij} = (\max(\mu_{j1}c(i), \lambda_{j1}c(i)), \min(0,0) (\max(\mu_{j1}c(i), \lambda_{j1}c(i), 0)$$

= $\max(\mu_{j1}c(i), \lambda_{j1}c(i))$, which is the define of addition(max) of two fuzzy matrice [2] in the usual sense three fuzzy reference is 0.

Definition : 2.5 [15]

Let A, BCFSM_{m×n}. Let the corresponding membership value, matrices be $M_v(A)=[\{\rho(A_{ij})\}_{m \times n}]$ and $M_v(B)=[\{\rho(B_{ij})\}_{m \times n}]$, $i=1,2,3,\dots,m$; then $j=1,2,3,\dots,n$. Then the score matrix $\rho(A,B)$ would be defined as $\rho(A,B)=[\rho_{ij}]_{m \times n}$ where $\rho_{ij} = \rho(A)_{ij} - \rho(B)_{ij}$.

III. Determinant of a fuzzy soft matrices

In this section, we start by introducing the nation of the determinant of a fuzzy soft matrices and we prove some properties. Also let S_n be the symmetric group on $\{1,2,3,\dots,n\}$.

Definition: 3:1

Let $A=[\mu_{ij}]_{m \times n}$ CFSM_{m×n} the determinant of A, denoted as $\det(A)$ or $|A|$, is defined as

$$|A| = \sum_{\pi \in S_n} \mu_{1\pi(1)} \mu_{2\pi(2)} \dots \mu_{n\pi(n)}$$

Where the summation is taken over all π of S_n .

Definition: 3:2

The determinant of the fuzzy soft matrix A of order 2 is denoted

$$|A| = \begin{vmatrix} (\mu_{11}c(1), \mu_{12}c(1) & (\mu_{21}c(1), \mu_{22}c(1)) \\ (\mu_{11}c(2), \mu_{12}c(2) & (\mu_{21}c(2), \mu_{22}c(2)) \end{vmatrix}$$

$$= [\max \{ \min(\mu_{11}c(1), \mu_{21}c(2)) \}, \min \{ \max(\mu_{21}c(1), \mu_{22}c(2)), \max(\mu_{22}c(1), \mu_{12}c(2)) \}].$$

Definition: 3.3

The determinant of the fuzzy soft matrix B order 3 is given by

$$|B| = \begin{vmatrix} (\lambda_{11}c(1), \lambda_{12}c(1)) & (\lambda_{21}c(1), \lambda_{22}c(1)) & (\lambda_{31}c(1), \lambda_{32}c(1)) \\ (\lambda_{11}c(2), \lambda_{12}c(2)) & (\lambda_{21}c(2), \lambda_{22}c(2)) & (\lambda_{31}c(2), \lambda_{32}c(2)) \\ (\lambda_{11}c(3), \lambda_{12}c(3)) & (\lambda_{21}c(3), \lambda_{22}c(3)) & (\lambda_{31}c(3), \lambda_{32}c(3)) \end{vmatrix}$$

$$|B| = (\lambda_{11}c(1), \lambda_{12}c(1)) \begin{vmatrix} (\lambda_{21}c(2), \lambda_{22}c(2)) & (\lambda_{31}c(2), \lambda_{32}c(2)) \\ (\lambda_{21}c(3), \lambda_{22}c(3)) & (\lambda_{31}c(3), \lambda_{32}c(3)) \end{vmatrix}$$

$$+ (\lambda_{21}c(1), \lambda_{22}c(1)) \begin{vmatrix} (\lambda_{11}c(2), \lambda_{12}c(2)) & (\lambda_{31}c(3), \lambda_{32}c(2)) \\ (\lambda_{11}c(3), \lambda_{12}c(3)) & (\lambda_{31}c(3), \lambda_{32}c(3)) \end{vmatrix}$$

$$+ (\lambda_{31}c(1), \lambda_{32}c(1)) \begin{vmatrix} (\lambda_{11}c(2), \lambda_{12}c(2)) & (\lambda_{21}c(2), \lambda_{22}c(2)) \\ (\lambda_{11}c(3), \lambda_{12}c(3)) & (\lambda_{21}c(3), \lambda_{22}c(3)) \end{vmatrix}$$

$$= (\lambda_{11}c(1), \lambda_{12}c(1)) [\max \{ \min((\lambda_{21}c(2), \lambda_{31}c(3)), \min((\lambda_{31}c(2), \lambda_{21}c(3))),$$

$$\begin{aligned}
 & \min \{ \max(\lambda_{22}(2), \lambda_{32}c(3)), \max(\lambda_{32}c(2), \lambda_{22}c(3)) \} \\
 & +(\lambda_{21}c(1), \lambda_{22}c(1)) [\max \{ \min((\lambda_{11}c(2), \lambda_{31}c(3)), \min((\lambda_{31}c(2), \lambda_{11}c(3))), \\
 & \qquad \min \{ \max(\lambda_{12}(2), \lambda_{32}c(3)), \max(\lambda_{32}c(2), \lambda_{12}c(3)) \}] \\
 & +(\lambda_{31}c(1), \lambda_{32}c(1)) [\max \{ \min((\lambda_{11}c(2), \lambda_{21}c(3)), \min((\lambda_{21}c(3), \lambda_{11}c(3))), \\
 & \qquad \min \{ \max(\lambda_{12}(2), \lambda_{22}c(3)), \max(\lambda_{22}c(2), \lambda_{13}c(3)) \}]
 \end{aligned}$$

Definition: 3:1

Let $U=\{c1,c2,c3\}$ be the universal set and E be the set of parameter given by $E=\{e1,e2,e3\}$ consider the fuzzy soft set.

$$F(e1) = \{c(c1,0.3,0.0), c(c2,0.5,0.1), c(c3,0.6,0.3)\},$$

$$F(e2) = \{c(c1,0.7,0.1), c(c2,0.9,0.5), c(c3,0.7,0.1)\},$$

$$F(e3) = \{c(c1,0.6,0.2), c(c2,0.7,0.0), c(c3,0.7,0.2)\}.$$

The fuzzy soft square matrix representing the fuzzy soft set is

$$A = \begin{vmatrix} (0.3,0.0) & (0.7,0.1) & (0.6,0.2) \\ (0.5,0.1) & (0.9,0.5) & (0.7,0.1) \\ (0.6,0.3) & (0.7,0.1) & (0.7,0.2) \end{vmatrix}$$

Then

$$\begin{aligned}
 |A| &= (0.3,0.0) \begin{vmatrix} (0.9,0.5) & (0.7,0.0) \\ (0.7,0.1) & (0.7,0.2) \end{vmatrix} + (0.7,0.1) \begin{vmatrix} (0.5,0.1) & (0.7,0.0) \\ (0.6,0.3) & (0.7,0.2) \end{vmatrix} \\
 &+ (0.6,0.2) \begin{vmatrix} (0.5,0.1) & (0.9,0.5) \\ (0.6,0.3) & (0.7,0.1) \end{vmatrix} \\
 &= (0.3,0.0) [\max \{ \min(0.9,0.7), \min(0.7,0.7) \}, \min \{ \max(0.5,0.2), \max(0.0,0.1) \}] \\
 &+ (0.7,0.1) [\max \{ \min(0.5,0.7), \min(0.7,0.6) \}, \min \{ \max(0.1,0.2), \max(0.0,0.3) \}] \\
 &+ (0.6,0.2) [\max \{ \min(0.5,0.7), \min(0.9,0.6) \}, \min \{ \max(0.1,0.1), \max(0.5,0.3) \}] \\
 &= (0.3,0.0)(0.7,0.1) + (0.7,0.1)(0.6,0.2) + (0.6,0.2)(0.6,0.1) \\
 &= [\max \{ \min(0.3,0.7) \}, \min \{ \max(0.0,0.1) \}] + \max \{ \min(0.7,0.6) \}, \\
 & \qquad \min \{ \max(0.1,0.2) \} + [\max \{ \min(0.6,0.6), \min \{ \max(0.2,0.1) \}] . \\
 &= (0.3,0.1) + (0.6,0.1) + (0.6,0.2)
 \end{aligned}$$

$$= \max(0.3,0.6,0.6)+\min(0.1,0.1,0.2)$$

$$|A| = (0.6,0.1)$$

Proportion:3.1

Let $ACFSM_{n \times n}$ then $\det A = \det A^T$

Proof:

Let $ACFSM_{2 \times 2}$ we write as

$$A = \begin{vmatrix} (\mu_{11c}(1), \mu_{12c}(1)) & (\mu_{21c}(1), \mu_{22c}(1)) \\ (\mu_{11c}(2), \mu_{12c}(2)) & (\mu_{21c}(2), \mu_{22c}(2)) \end{vmatrix}$$

$$A^T = \begin{vmatrix} (\mu_{11c}(1), \mu_{12c}(1)) & (\mu_{11c}(2), \mu_{12c}(2)) \\ (\mu_{21c}(2), \mu_{22c}(2)) & (\mu_{21c}(1), \mu_{22c}(1)) \end{vmatrix}$$

$$\det A^T = [\max\{\min(\mu_{11c}(1), \mu_{12c}(2)), \min(\mu_{11c}(2), \mu_{21c}(1))\}$$

$$\text{Min}\{\max(\mu_{12c}(1), \mu_{22c}(2)), \max(\mu_{12c}(2), \mu_{22c}(1))\}$$

$$= [\max\{\min((\mu_{11c}(1), \mu_{21c}(2)), \min((\mu_{21c}(1), \mu_{11c}(2))),$$

$$\text{Min}\{(\mu_{12c}(1), \mu_{22c}(2)), \max(\mu_{22c}(1), \mu_{12c}(2))\}]$$

$$= \det[A].$$

Example : 3.2:

$$\text{Let } A = \begin{vmatrix} (0.8,0) & (0.3,0) & (0.2,0) \\ (0.6,0) & (0.9,0) & (0.6,0) \\ (0.1,0) & (0.7,0) & (0.7,0) \end{vmatrix}$$

$$A^T = \begin{vmatrix} (0.8,0) & (0.6,0) & (0.1,0) \\ (0.3,0) & (0.9,0) & (0.7,0) \\ (0.2,0) & (0.6,0) & (0.7,0) \end{vmatrix}$$

$$\det [A] = (0.8,0) \begin{vmatrix} (0.9,0) & (0.6,0) \\ (0.7,0) & (0.7,0) \end{vmatrix} + (0.3,0) \begin{vmatrix} (0.6,0) & (0.6,0) \\ (0.1,0) & (0.7,0) \end{vmatrix} + (0.2,0) \begin{vmatrix} (0.6,0) & (0.9,0) \\ (0.1,0) & (0.7,0) \end{vmatrix}$$

$$= (0.8,0)[(0.7,0)+(0.6,0)] + (0.30)[(0.6,0)+(0.1,0)] + (0.2,0)[(0.6,0)+(0.1,0)]$$

$$= (0.7,0) + (0.3,0) + (0.2,0)$$

$$\det[A] = (0.7,0)$$

$$\det A^T = \begin{vmatrix} (0.8,0) & (0.6,0) & (0.1,0) \\ (0.3,0) & (0.9,0) & (0.6,0) \\ (0.1,0) & (0.7,0) & (0.7,0) \end{vmatrix}$$

$$\begin{aligned}
 & (0.8,0) \begin{vmatrix} (0.9,0) & (0.6,0) \\ (0.7,0) & (0.7,0) \end{vmatrix} + (0.6,0) \begin{vmatrix} (0.3,0) & (0.6,0) \\ (0.1,0) & (0.7,0) \end{vmatrix} + (0.1,0) \begin{vmatrix} (0.3,0) & (0.9,0) \\ (0.1,0) & (0.7,0) \end{vmatrix} \\
 &= (0.8,0)[(0.7,0)+(0.6,0)] + (0.60)[(0.3,0)+(0.1,0)] + (0.1,0)[(0.3,0)+(0.1,0)] \\
 &= (0.7,0) + (0.3,0) + (0.1,0)
 \end{aligned}$$

$$\det A^T = (0.7,0)$$

∴ hence $\det[A]^T = \det [A]$

Proposition : 3.2

Let aCFSM, then $\det[aA] = a \det[A]$

Proof :

Let aCFSM, and $A = [\mu_{ij}] \text{ CFSM}_{2 \times 2}$

$$\begin{aligned}
 \det[aA] &= \begin{vmatrix} a(\mu_{11}(c1), \mu_{12}(c1)) & a(\mu_{11}(c2), \mu_{12}(c2)) \\ a(\mu_{21}(c1), \mu_{22}(c1)) & a(\mu_{21}(c2), \mu_{22}(c2)) \end{vmatrix} \\
 &= a \begin{vmatrix} (\mu_{11}(c1), \mu_{12}(c1)) & (\mu_{11}(c2), \mu_{12}(c2)) \\ (\mu_{21}(c1), \mu_{22}(c1)) & (\mu_{21}(c2), \mu_{22}(c2)) \end{vmatrix} \\
 &= a \det[A]
 \end{aligned}$$

Example 3:3

$$\text{if } A = \begin{vmatrix} 0.8 & 0.3 & 0.2 \\ 0.6 & 0.9 & 0.6 \\ 0.1 & 0.7 & 0.7 \end{vmatrix}$$

$$\begin{aligned}
 \det [A] &= 0.8 \begin{vmatrix} 0.9 & 0.6 \\ 0.7 & 0.7 \end{vmatrix} + 0.3 \begin{vmatrix} 0.6 & 0.6 \\ 0.1 & 0.7 \end{vmatrix} + 0.2 \begin{vmatrix} 0.6 & 0.9 \\ 0.1 & 0.7 \end{vmatrix} \\
 &= 0.8 [0.7 + 0.6] + 0.3[0.6+0.1] + 0.2[0.6+0.1] \\
 &= 0.7 + 0.3 + 0.2
 \end{aligned}$$

$$\det[A] = 0.7$$

Set $a = 0.5$

$$aA = 0.5 \begin{vmatrix} 0.8 & 0.3 & 0.2 \\ 0.6 & 0.9 & 0.6 \\ 0.1 & 0.7 & 0.7 \end{vmatrix}$$

$$aA = \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.5 & 0.5 & 0.5 \\ 0.1 & 0.5 & 0.5 \end{bmatrix}$$

$$\det[aA] = 0.5(0.5) + 0.3 + 0.2(0.5)$$

$$\det[aA] = 0.5$$

$$a \det[A] = (0.5)(0.7) = 0.5$$

$$\text{hence } |aA| = a|A| = 0.5$$

$$\text{(or) } \det[aA] = a \det[A]$$

IV. Properties of Determinant ordering with fuzzy soft matrices

Definition 4.1

The determinant ordering $A \leq B$ in $M_n(Fs)$ is defined as $\tilde{A} \leq \tilde{B} \Leftrightarrow \det[\tilde{A}] \leq \det[\tilde{B}]$. (or) $A \leq \tilde{B} \Leftrightarrow |\tilde{A}| \leq |\tilde{B}|$.

Example 4:1

$$\text{Let } \tilde{A} = \begin{bmatrix} (0.5,0.1) & (0.4,0.2) & (0.6,0.2) \\ (0.5,0.1) & (0.4,0.2) & (0.6,0.2) \\ (0.5,0.2) & (0.2,0.1) & (0.5,0.1) \end{bmatrix}$$

$$\text{a and } \tilde{B} = \begin{bmatrix} (0.7,0.2) & (0.6,0.3) & (0.8,0.4) \\ (0.4,0.5) & (0.5,0.3) & (0.6,0.4) \\ (0.8,0.3) & (0.6,0.2) & (0.7,0.3) \end{bmatrix}$$

$$\begin{aligned} \det[\tilde{A}] &= (0.5,0.1) [\max(0.1,0.2), \min(0.2,0.3)] + \\ &\quad (0.4,0.2) [\max(0.3,0.4), \min(0.3,0.3)] + \\ &\quad (0.6,0.2) [\max(0.2,0.1), \min(0.2,0.3)] \\ &= (0.5,0.1) (0.2,0.2) + (0.4,0.2) (0.4,0.3) + (0.6,0.2) (0.2,0.2) \\ &= (0.2,0.2) + (0.4,0.3) + (0.2,0.2) \end{aligned}$$

$$\det[\tilde{A}] = (0.4,0.2)$$

$$\begin{aligned} \det[\tilde{B}] &= (0.7,0.2) [\max(0.5,0.6), \min(0.3,0.4)] + \\ &\quad (0.6,0.3) [\max(0.4,0.6), \min(0.5,0.4)] + \\ &\quad (0.8,0.4) [\max(0.4,0.5), \min(0.5,0.3)] \\ &= (0.7,0.2) (0.6,0.3) + (0.6,0.3) (0.6,0.4) + (0.8,0.4) (0.5,0.3) \\ &= (0.6,0.3) + (0.6,0.4) + (0.5,0.4) \end{aligned}$$

$$\det[\tilde{B}] = (0.6, 0.3)$$

Hence $A \leq B \Leftrightarrow \det[A] \leq \det[B]$.

Theorem 4.1

The determinant ordering is not a partial ordering.

Proof:

(i) $\det[\tilde{A}] = \det[\tilde{B}]$ for all $\tilde{A} \in M_n(\text{FS})$

Hence $\tilde{A} \leq \tilde{B}$

\therefore reflexivity is true.

(ii) $\tilde{A} \leq \tilde{B} \Rightarrow \det[\tilde{A}] \leq \det[\tilde{B}]$,

$\tilde{B} \leq \tilde{A} \Rightarrow \det[\tilde{B}] \leq \det[\tilde{A}]$,

$\tilde{A} \leq \tilde{B}$ and $\tilde{B} \leq \tilde{A} \Rightarrow \det[\tilde{A}] = \det[\tilde{B}]$.

But $\det[\tilde{A}] = \det[\tilde{B}]$

\therefore does not imply $\tilde{A} = \tilde{B}$

\therefore Anti Symmetry is not true.

(iii) $\tilde{A} \leq \tilde{B}, \tilde{B} \leq C \Rightarrow \tilde{A} \leq C$ for all $\tilde{A}, \tilde{B}, C \in M_n(\text{Fs})$

For $\tilde{A} \leq \tilde{B} \Rightarrow \det[\tilde{A}] \leq \det[\tilde{B}]$

$\tilde{B} \leq C \Rightarrow \det[\tilde{B}] \leq \det[C]$

$\tilde{A} \leq C \Rightarrow \det[\tilde{A}] \leq \det[C]$

\therefore transitivity is true.

Thus the determinant ordering is not a partial ordering in $M_n(\text{Fs})$

Example 4.2

$$\text{Let } \tilde{A} = \begin{vmatrix} (0.5, 0.1) & (0.4, 0.2) & (0.6, 0.2) \\ (0.3, 0.3) & (0.1, 0.2) & (0.4, 0.3) \\ (0.5, 0.2) & (0.2, 0.1) & (0.5, 0.1) \end{vmatrix}$$

$$\tilde{B} = \begin{vmatrix} (0.7, 0.2) & (0.6, 0.3) & (0.8, 0.4) \\ (0.4, 0.5) & (0.5, 0.3) & (0.6, 0.4) \\ (0.8, 0.3) & (0.6, 0.2) & (0.7, 0.3) \end{vmatrix}$$

$$\text{and } C = \begin{vmatrix} (0.8, 0.3) & (0.6, 0.4) & (0.8, 0.5) \\ (0.5, 0.6) & (0.9, 0.4) & (0.7, 0.5) \\ (0.8, 0.4) & (0.6, 0.4) & (0.8, 0.5) \end{vmatrix}$$

$$\det[\tilde{A}] = (0.4, 0.2), \det[\tilde{B}] = (0.6, 0.3)$$

$$\det[C] = (0.8, 0.3) (0.8, 0.5) + (0.6, 0.4) (0.7, 0.5) + (0.8, 0.5) (0.5, 0.4)$$

$$= (0.8,0.5) + (0.6,0.5) + (0.5,0.5)$$

$$= (0.8,0.5)$$

(i) $\det[\tilde{A}] \leq \det[\tilde{A}]$ for all $\tilde{A} \in M_n(F_s)$, $\tilde{A} \leq \tilde{A}$,

∴ reflexivity is true.

(ii) $\tilde{A} \leq \tilde{B} \Rightarrow \det[\tilde{A}] \leq \det[\tilde{B}] = (0.4,0.2) \leq (0.6,0.3)$

$$\tilde{B} \leq \tilde{A} \Rightarrow \det[\tilde{B}] \leq \det[\tilde{A}] = (0.6,0.3) \leq (0.4,0.2)$$

$$\tilde{A} \leq \tilde{B} \text{ and } \tilde{B} \leq \tilde{A} \Rightarrow \det[\tilde{A}] = \det[\tilde{B}]$$

But $\det[\tilde{A}] = \det[\tilde{B}]$ does not imply $\tilde{A} = \tilde{B}$

∴ Anti symmetry is not true

(iii) $\tilde{A} \leq \tilde{B} = \det[\tilde{A}] \leq \det[\tilde{B}] = (0.4,0.2) \leq (0.6,0.3)$

$$\tilde{B} \leq C = \det[\tilde{B}] \leq \det[C] = (0.6,0.3) \leq (0.8,0.5)$$

$$\tilde{A} \leq C = \det[\tilde{A}] \leq \det[C] = (0.4,0.2) \leq (0.8,0.5)$$

∴ Transitivity is true.

Thus the determinant ordering is not a partial ordering in $M_n(F_s)$.

V. Conclusion

In this paper, we introduce definition of determinant on fuzzy soft matrix and its properties are discussed. Numerical examples are given to clarify the developed theory and the proposed determinant of a square fuzzy soft matrices.

Reference

- [1] Bertoinuzza, On the distributivity of t-norm an t-c norms, pr, 2n IEEE internal. On fuzzy system(IEEE press, Piscataway, NJ, 1993) 14 – 17.
- [2] Jian Mia Chen. (1982). “Fuzzy matrix partial ordering an generalize inverse.” Fuzzy sets Systems. 1 5:453 – 458
- [3] Meenakshi A.R. an C. Kilavany R., On fuzzy 2-norme linear Spaces, the Journal of fuzzy mathematics, volume 9(N .2) 2 1 (345 – 351).
- [4] Nag rgani A. an Manikan an A. R. On Fuzzy 2-norme fuzzy matrices. J. Math. Comput. Sci. 3 (2 13), N .1, 233-241, ISSN : 1927-53 7
- [5] Nag rgani A. an Kalyani G. Binorme sequences in fuzzy matrices. Bul- letin of Pure an Applie Science. V 1. 22E(N .2) 2 3; P.445-451
- [6] Nag rgani A. an Kalyani G. “Fuzzy matrix m- ordering.” Science Letters, v 1,27 N ,3 an 4,2 4.
- [7] Raga .M. Z an Emam E. G. The deteminant an a joint f a square fuzzy matrix, An international Journal f Information Sciences – Intelligent Systems, V 1 84, 1995, 2 9 – 22
- [8] L.A.Za eh, Fuzzy Sets, Information an Control 8 (1995) 338{353}
- [9] ZHOU Min – na “Characterizations of the Minus Ordering in Fuzzy Matrix Set” Journal foning university (onsee) V 1.21 N .4 Dec .28