

A NEW OPERATIONS ON BIPOLAR INTUITIONISTIC FUZZY GRAPHS

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ABSTRACT

In this paper, bipolar Intuitionistic fuzzy graphs with four operations namely Cartesian product, composition, tensor product, normal product are defined. Also, the degrees of the vertices of the resultant graphs which are obtained from two given bipolar intuitionistic fuzzy graphs G_1 and G_2 using the operations union, join, intersection ring sum, direct product and Cartesian product and complement on bipolar fuzzy graphs.

KEYWORDS: Bipolar Intuitionistic fuzzy graph, union, join and complement, intersection ring sum, direct product and Cartesian product.

INTRODUCTION

In 1965, Zadeh [21] represented the uncertainly as fuzzy subset of sets. Since than the theory of fuzzy sets has become a vigorous area of research in different disciplines including medical and life sciences, management sciences, social sciences, engineering, statistics, graph theory, artificial intelligence, signal processing, meeltiagent systems, pattern recognition, robotics, computer networks, expert systems, decision making, automata theory, etc. Graph theory has numerous applications to problems in computer science, networking routing, system analysis, electrical engineering, operations research, economics, transportation and many others. In many cases some aspects of a graph theoretic problem may be uncertain. The bipolar fuzzy sets have been explained by zhang [22] in 1994. Zhang extended the fuzzy sets as bipolar fuzzy sets by assigning the membership value in the range $[-1,1]$. In a bipolar fuzzy set, the membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree $[0,1]$ of an element indicates that the element somewhat satisfies the property, and the membership degree $[-1,0]$ of an element indicates the element somewhat satisfies the implicit counter property.

In 2001, Mordeson and Nair [3] discussed about the properties of fuzzy graphs and hypergraphs. After that, the operation of union, join, Cartesian product and composition on two fuzzy graphs was defined by Mordeson and peng [4]. Bhatlacharya in 1987 developed some remarks on fuzzy graphs. Hai Long yang et al. [2] gave the generalized bipolar fuzzy graphs. Atanassov [1] introduced the concepts of intuitionistic fuzzy set as a generalization of fuzzy sets. Atanassov added a new component which determines the degree of non-membership in the definition of fuzzy set. In 1975, Rosenfeld [9] discussed the concepts of fuzzy graphs. The fuzzy relations between fuzzy sets were also considered by Rosenfeld and he developed the structure of fuzzy graph, obtained analogs of several graphs theoretical concepts. After Rosenfeld [9] the fuzzy graph theory increases with its various types of branches, such as – fuzzy tolerance graph [16], fuzzy threshold graph [15], bipolar fuzzy graphs and [7,8,19], balanced interval-valued fuzzy graphs [5,6], fuzzy K-Competition fuzzy planar graphs [14,20], bipolar fuzzy hypergraphs [17], etc. also several works have been done on fuzzy graphs by Samata and Pal[19s]. Sahoo and pal[11] discussed the concept of intuitionistic fuzzy competition graph. They also discussed intuitionistic fuzzy tolerance graph with application [12], different types of products on intuitionistic fuzzy graph [10] and product of intuitionistic fuzzy graph and their degrees [13].In this paper,bipolar intuitionstic fuzzy graphs with four operations namely Cartesian product, Composition, Tensor product,normal product are defined. Also, the degrees of the vertices of the resultant graphs which ae obtained from two given bipolar intuitionistic fuzzy graphs G_1 and G_2 usng the operations union,join,intersection rng sum, direct product and Cartesian product and complement on bipolar fuzzy graphs.

PRELIMINARIES

Let v be a universe of discourse it may be taken as the set of vertices of a graphs G . If the membership value of $u \in v$ is non-zero, then u considered as a vertex of G .

Definition:2.1

Let $G = (V, \sigma, \mu)$ be a fuzzy graph, the degree of a vertex u in G is defined by,

$$Du = \sum_{u \neq v} \mu(u,v) = \sum_{uv \in E} \mu(u,v)$$

Definition:2.2

A bipolar fuzzy graph with an underlying set V is defined to be a pair $G = (V, A, B)$

Where $A = (\mu_A^P, \mu_A^N)$ is a bipolar fuzzy set in V and $B = (\mu_B^P, \mu_B^N)$ is a bipolar fuzzy set in $E \subseteq V \times V$ such that $\mu_B^P(x,y) \leq \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N(x,y) \geq \max(\mu_A^N(x), \mu_A^N(y))$ for all $(x,y) \in E$.

Definition:2.3

An intuitionistic fuzzy graph is of the form $G = (V, \mu, \lambda)$ where

- (i) The vertex set $V = \{v_0, v_1, v_2, \dots, v_n\}$ such that $\mu_1 : V \rightarrow [0,1]$ and $\lambda_1 : V \rightarrow [0,1]$ denote the degree of membership and non-membership of the vertex $v_i \in V$ ($i=1,2,3, \dots, n$) and
- (ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \rightarrow [0,1]$ and $\lambda_2 : V \times V \rightarrow [0,1]$ where $\mu_2(v_i, v_j)$ and $\lambda_2(v_i, v_j)$ denote the degree of membership and non-membership value of the edge (v_i, v_j) respectively such that $\mu_2(v_i, v_j) \leq \min\{\mu_1(v_i), \mu_1(v_j)\}$ and $\lambda_2(v_i, v_j) \geq \max\{\lambda_1(v_i), \lambda_1(v_j)\}$, $0 \leq \mu_2(v_i, v_j) + \lambda_2(v_i, v_j) \leq 1$ for every edge (v_i, v_j) .

The main objective of this paper is to study the bipolar intuitionistic fuzzy graph and this graph is based on the bipolar intuitionistic fuzzy set defined below.

Definition:2.4

A dombi bipolar fuzzy graph $G = (V, A, B)$ is a non-empty finite set on V together with a pair of functions

$A = (\mu_A^P, \mu_A^N) : V \rightarrow [0,1] \times [-1,0]$ and $B = (\mu_B^P, \mu_B^N) : V \times V \rightarrow [0,1] \times [-1,0]$ such that for every $x, y \in V$.

$$B_1^P(xy) \leq \frac{A_1^P(x)A_1^P(y)}{A_1^P(x) + A_1^P(y) - A_1^P(x)A_1^P(y)} \quad \text{and}$$

$$B_1^N(xy) \geq \frac{A_1^N(x)A_1^N(y)}{A_1^N(x) + A_1^N(y) - A_1^N(x)A_1^N(y)}$$

Then one can say that $A = (A_1^P, A_1^N)$ as the demobi bipolar vertes set of G and $B = (B_1^P, B_1^N)$ as the dombi bipolar edge set of G .

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Definition: 3.1

Let A be the bipolar fuzzy subset of V_i and B_i be the fuzzy subset of $E_i = i=1, 2, \dots$ define the direct product of $(G_1 \times G_2) = (\mu_1 \times \mu_2, \lambda_1 \times \lambda_2)$ of the intuitionistic bipolar fuzzy graphs $G_1 = (\mu_1, \lambda_1)$ and $G_2 = (\mu_2, \lambda_2)$ respectively given as follows.

$$(i) (\mu_1^P, \mu_2^P)(x_1, x_2) = \frac{\mu_1^P(x_1) \mu_2^P(x_2)}{\mu_1^P(x_1) + \mu_2^P(x_2) - \mu_1^P(x_1) \mu_2^P(x_2)} \text{ for}$$

Every $(x_1, x_2) \in V_1 \times V_2$ and

$$(\mu_1^N, \mu_2^N)(x_1, x_2) = \frac{\mu_1^N(x_1) \mu_2^N(x_2)}{\mu_1^N(x_1) + \mu_2^N(x_2) - \mu_1^N(x_1) \mu_2^N(x_2)} \text{ for}$$

Every $(x_1, x_2) \in V_1 \times V_2$ and

$$(ii) (\lambda_1^P, \lambda_2^P)(x_1, x_2), (y_1, y_2) = \frac{\lambda_1^P(x_1 y_1) \lambda_2^P(x_2 y_2)}{\lambda_1^P(x_1 y_1) + \lambda_2^P(x_2 y_2) - \lambda_1^P(x_1 y_1) \lambda_2^P(x_2 y_2)} \text{ for}$$

Every $(x_1 x_2, y_1 y_2) \in E_1 \times E_2$ and

$$(\lambda_1^N, \lambda_2^N)(x_1, x_2), (y_1, y_2) = \frac{\lambda_1^N(x_1 y_1) \lambda_2^N(x_2 y_2)}{\lambda_1^N(x_1 y_1) + \lambda_2^N(x_2 y_2) - \lambda_1^N(x_1 y_1) \lambda_2^N(x_2 y_2)} \text{ for}$$

Every $(x_1 x_2, y_1 y_2) \in E_1 \times E_2$.

Definition:3.2

Let μ_i be a bipolar fuzzy subset of V_i and let λ_i be a fuzzy subset of $E_i, i=1,2,\dots$

Define the union

ac (0.4,-0.046) ad (0.3,-0.17) ae(0.35,-0.166)

(0.20, -0.046)	(0.20,-0.05)
(0.18, -0.13)	(0.2,-0.13)

(0.20,-0.046)

bc(0.3,-0.19)

bd(0.25,-0.2)

be(0.3,-0.17)

$G_1 \square G_2$

Cartesian product of intuitionistic bipolar fuzzy graph.

$G_1 \cup G_2 = (\mu_1 \cup \mu_2), (\lambda_1 \cup \lambda_2)$ of intuitionistic bipolar fuzzy graph.

$G_1=(\mu_1, \lambda_1)$ and $G_2=(\mu_2, \lambda_2)$ be given as follows:

$$(\mu_1 \cup \mu_2)^P(x) = \begin{cases} \mu_1^P(x) & \text{if } x \in V1 \setminus V2 \\ \mu_2^P(x) & \text{if } x \in V2 \setminus V1 \\ \frac{\mu_1^P(x) \mu_2^P(y)}{\mu_1^P(x) + \mu_2^P(y) - \mu_1^P(x) \mu_2^P(y)} & \text{if } x \in V1 \cap V2 \end{cases}$$

$$(\mu_1 \cup \mu_2)^N(x) = \begin{cases} \mu_1^N(x) & \text{if } x \in V1 \setminus V2 \\ \mu_2^N(x) & \text{if } x \in V2 \setminus V1 \\ \frac{\mu_1^N(x) \mu_2^N(y)}{\mu_1^N(x) + \mu_2^N(y) - \mu_1^N(x) \mu_2^N(y)} & \text{if } x \in V1 \cap V2 \end{cases}$$

Also

$$(\lambda_1 \cup \lambda_2)^P(xy) = \begin{cases} \lambda_1^P(xy) / 2 & \text{if } xy \in E_1 \setminus E_2 \\ \lambda_2^P(xy) / 2 & \text{if } xy \in E_2 \setminus E_1 \\ \frac{\mu_1^P(x)\mu_2^P(y)}{\mu_1^P(x)+\mu_2^P(y)-\mu_1^P(x)\mu_2^P(y)} / 2 & \text{if } xy \in E_1 \cap E_2 \end{cases}$$

$$(\lambda_1 \cup \lambda_2)^N(xy) = \begin{cases} \lambda_1^N(xy) / 2 & \text{if } xy \in E_1 \setminus E_2 \\ \lambda_2^N(xy) / 2 & \text{if } xy \in E_2 \setminus E_1 \\ \frac{\mu_1^N(x)\mu_2^N(y)}{\mu_1^N(x)+\mu_2^N(y)-\mu_1^N(x)\mu_2^N(y)} / 2 & \text{if } xy \in E_1 \cap E_2 \end{cases}$$

Example:

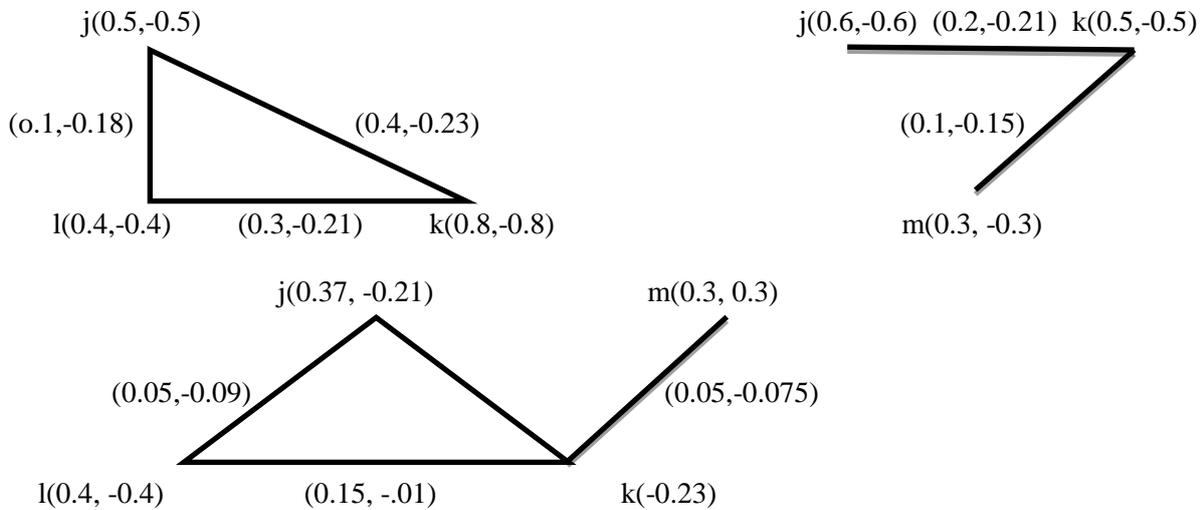
The intuitionistic fuzzy graphs are

$G_1 = \{ j (0.5, -0.5), k (0.8,-0.8), l (0.4,-0.4) \}$ and

$G_2 = \{ j (0.6,-0.6), k (0.5, -0.5), m (0.3,-0.3) \}$. Then

$$\text{We have } \mu_1 \cup \mu_2 = \frac{j}{(0.37,-0.21)} \cup \frac{k}{(0.4,-0.23)} \cup \frac{l}{(0.4,-0.4)} \cup \frac{m}{(0.3,-0.3)}$$

$$\lambda_1 \cup \lambda_2 = \frac{jl}{(0.05,-0.09)} \cup \frac{jk}{(0.11,-0.09)} \cup \frac{mk}{(0.05,-0.075)} \cup \frac{lk}{(0.15,-0.1)}$$



Theorem: 3.3

The union $G_1 \cup G_2$ of G_1 and G_2 is the intuitionistic bipolar fuzzy graph of G if and only if G_1 and G_2 are the intuitionistic bipolar fuzzy graphs of G_1 where μ_1, μ_2, λ_1 and λ_2 are bipolar subsets of V_1, V_2, E_1 and E_2 respectively, provided $V_1 \cap V_2 = \emptyset$.

Proof:

Assume that $G_1 \cup G_2$ are intuitionistic bipolar fuzzy graphs. Let $xy \in E_1$, then $xy \in E_2$ and $x, y \in V_1 \setminus V_2$. Then

$$\lambda_1^P(xy) = (\lambda_1 \cup \lambda_2)^P(xy)$$

$$\leq [(\mu_1^P \cup \mu_2^P)(x) (\mu_1^P \cup \mu_2^P)(y)] / [(\mu_1^P \cup \mu_2^P)(x) + (\mu_1^P \cup \mu_2^P)(y) - (\mu_1^P \cup \mu_2^P)(x) (\mu_1^P \cup \mu_2^P)(y)]$$

$$\lambda_1^P(xy) \leq \frac{\mu_1^P(x)\mu_2^P(y)}{\mu_1^P(x)+\mu_2^P(y)-\mu_1^P(x)\mu_2^P(y)}$$

Thus, G_1 is the intuitionistic bipolar fuzzy graph of G . similarly it is easy to verify that G_2 the intuitionistic bipolar fuzzy graph of G . conversely, assume that G_1 and G_2 are the intuitionistic bipolar fuzzy graph of G respectively consider $xy \in E_1 \setminus E_2$, then by the definition of union, it follows that,

$$(\lambda_1 \cup \lambda_2)^P(xy) = \lambda_1^P(xy) \leq (\mu_1^P(x), \mu_1^P(y)) = ((\mu_1 \cup \mu_2)^P(x), (\mu_1 \cup \mu_2)^P(x))$$

In the similar way, we can find $xy \in E_2 \setminus E_1$

$$\begin{aligned} (\lambda_1 \cup \lambda_2)^P(xy) &\leq ((\mu_1 \cup \mu_2)^P(x), (\mu_1 \cup \mu_2)^P(y)) \\ &\leq ((\mu_1 \cup \mu_2)^P(x), (\mu_1 \cup \mu_2)^P(y)) / [(\mu_1 \cup \mu_2)^P(x) + (\mu_1 \cup \mu_2)^P(y) - (\mu_1 \cup \mu_2)^P(x), (\mu_1 \cup \mu_2)^P(y)] \\ &\leq \frac{(\mu_1 \cup \mu_2)^P(x), (\mu_1 \cup \mu_2)^P(y)}{(\mu_1 \cup \mu_2)^P(x) + (\mu_1 \cup \mu_2)^P(y) - (\mu_1 \cup \mu_2)^P(x), (\mu_1 \cup \mu_2)^P(y)} \end{aligned}$$

Also

$$\begin{aligned} \lambda_1^N(xy) &\leq \frac{\mu_1^N(x) \mu_1^N(y)}{\mu_1^N(x) + \mu_1^N(y) - \mu_1^N(x) \mu_1^N(y)} \\ (\lambda_1 \cup \lambda_2)^N(xy) &\leq \frac{(\mu_1 \cup \mu_2)^N(x), (\mu_1 \cup \mu_2)^N(y)}{(\mu_1 \cup \mu_2)^N(x) + (\mu_1 \cup \mu_2)^N(y) - (\mu_1 \cup \mu_2)^N(x), (\mu_1 \cup \mu_2)^N(y)} \end{aligned}$$

Definition: 3.4

Let μ_i be a bipolar fuzzy subset of V_i and let λ_i be a bipolar fuzzy subset of E_i , $i=1,2,\dots$. Defined the ring sum $G_1 \oplus G_2 = (\mu_1 \oplus \mu_2, \lambda_1 \oplus \lambda_2)$ of the intuitionistic bipolar fuzzy graphs, $G_1=(\mu_1, \lambda_1)$ and $G_2=(\mu_2, \lambda_2)$ satisfies the following conditions.

(i) $(\mu_1 \oplus \mu_2)^P(x) = (\mu_1 \cup \mu_2)^P(x)$ if $x \in V_1 \cup V_2$

(ii) $(\mu_1 \oplus \mu_2)^N(x) = (\mu_1 \cup \mu_2)^N(x)$ if $x \in V_1 \cup V_2$

(iii) $(\lambda_1 \oplus \lambda_2)^P(xy) = \begin{cases} \lambda_1^P(xy) & \text{if } xy \in E_1 \setminus E_2 \\ \lambda_2^P(xy) & \text{if } xy \in E_2 \setminus E_1 \\ 0 & \text{if } xy \in E_1 \cap E_2 \end{cases}$

(iv) $(\lambda_1 \oplus \lambda_2)^N(xy) = \begin{cases} \lambda_1^N(xy) & \text{if } xy \in E_1 \setminus E_2 \\ \lambda_2^N(xy) & \text{if } xy \in E_2 \setminus E_1 \\ 0 & \text{if } xy \in E_1 \cap E_2 \end{cases}$

Theorem:3.5

The ring sum $G_1 \oplus G_2$ two bipolar intuitionistic fuzzy graphs G_1 and G_2 of G_1^* and G_2^* is the intuitionistic bipolar fuzzy graph of $G_1 \oplus G_2$.

Proof:

Consider $xy \in E_1 \setminus E_2$ then there are three possibilities such as

(i) $x, y \in V_1 \setminus V_2$

(ii) $x \in V_1 \setminus V_2, y \in V_1 \cap V_2$

(iii) $x, y \in V_1 \cap V_2$

i) suppose $x, y \in V_1 \setminus V_2$ then by the definition of ring sum we have

$$\begin{aligned} (\lambda_1 \oplus \lambda_2)^P(xy) &= \lambda_1^P(xy) \\ &\leq (\mu_1^P(x), \mu_1^P(y)) \\ &\leq ((\mu_1 \cup \mu_2)^P(x), (\mu_1 \cup \mu_2)^P(y)) \end{aligned}$$

$$\begin{aligned} &\leq ((\mu_1 \cup \mu_2)^P(x), (\mu_1 \cup \mu_2)^P(y)) \\ \text{ii) suppose } x \in V_1 \setminus V_2, y \in V_1 \cap V_2 \\ (\lambda_1 \oplus \lambda_2)^P(xy) &= \lambda_1^P(xy) \\ &\leq (\mu_1^P(x), \mu_1^P(y)) \\ &\leq ((\mu_1 \cup \mu_2)^P(x), (\mu_1^P(y))) \\ &\leq ((\mu_1 \oplus \mu_2)^P(x), (\mu_1^P(y))) \end{aligned}$$

Clearly,

$$\mu_1(y) \leq \frac{\mu_1(y) + \mu_2(y) - 2\mu_1(y)\mu_2(y)}{1 - \mu_1(y)\mu_2(y)}$$

by substituting $A_1(y)=f, A_2(y)=h$
we have

$$f \leq \frac{f+h-2fh}{1-fh}$$

$$\begin{aligned} f-f^2h &\leq f+h-2fh \\ 0 &\leq (f-1)^2 \end{aligned}$$

since

$$(\lambda_1 \oplus \lambda_2)^P(xy) \leq ((\mu_1 \oplus \mu_2)(x), (\mu_1 \oplus \mu_2)(y))$$

iii) suppose $x, y \in V_1 \cap V_2$, then we get

$$\begin{aligned} (\lambda_1 \oplus \lambda_2)(xy) &= \lambda_1(xy) \\ &\leq \mu_1(x), \mu_1(y) \\ &\leq ((\mu_1 \cup \mu_2)(x), (\mu_1 \cup \mu_2)(y)) \\ &\leq ((\mu_1 \oplus \mu_2)(x), (\mu_1 \oplus \mu_2)(y)) \end{aligned}$$

Since by (ii) we have

$$\begin{aligned} \mu_1(x) &\leq \frac{\mu_1(x) + \mu_2(x) - 2\mu_1(x)\mu_2(x)}{1 - \mu_1(x)\mu_2(x)} \text{ and} \\ \mu_1(y) &\leq \frac{\mu_1(y) + \mu_2(y) - 2\mu_1(y)\mu_2(y)}{1 - \mu_1(y)\mu_2(y)} \end{aligned}$$

because of symmetry, we obtain for $xy \in E_2 \setminus E_1$ in the three possible cases

$$\begin{aligned} (\lambda_1 \oplus \lambda_2)^P(xy) &\leq ((\mu_1 \oplus \mu_2)^P(x), (\mu_1 \oplus \mu_2)^P(y)) \\ &\leq [(\mu_1 \oplus \mu_2)^P(x) (\mu_1 \oplus \mu_2)^P(y)] \\ &\quad - [(\mu_1 \oplus \mu_2)^P(x) + (\mu_1 \oplus \mu_2)^P(y) - (\mu_1 \oplus \mu_2)^P(x) (\mu_1 \oplus \mu_2)^P(y)] \end{aligned}$$

Definition:3.6

A bipolar fuzzy graph $G=(A, B)$ of a graph $G^* = (V, E)$ is said to be self-weak complement if G is weak isomorphism with its complement \bar{G} , i.e. there exist a bijective homomorphism f from G to \bar{G} such that for all $x, y \in V$.

$$\begin{aligned} \mu_A^P(x) &= \overline{\mu_A^P}(f(x)), \mu_A^N(x) = \overline{\mu_A^N}(f(x)) \text{ and} \\ \mu_B^P(xy) &\leq \overline{\mu_B^P}(f(x) f(y)), \mu_B^N(xy) \leq \overline{\mu_B^N}(f(x) f(y)) \end{aligned}$$

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