

An Application Of Decision Making Problem By Using Adjoint Of A Square Fuzzy Soft Matrices

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Abstract : In this paper we introduce fuzzy adjoint ordering with fuzzy soft matrices using the structure of $M_n(FS)$ the set of $(n \times n)$ fuzzy adjoint ordering with fuzzy soft matrices is introduce and its applied adjoint of fuzzy soft matrix in decision making problem

keywords: Soft set, fuzzy soft set (FSS), fuzzy soft Matrix (FSM), determinant of a square fuzzy soft matrix ,adjoint of a fuzzy soft matrices.

I. Introduction

The concept of fuzzy set was introduce by Zabehe [8] in 1965. Jian Miao chen [2] introduced the fuzzy matrix partial ordering and generalized inverse. Bertoiuzza [1] introduced the distributivity of t – norm and t – conforms. In 1995, Ragab. M. Z and Emam E.G. [3] introduced the determinant and adjoint of a square fuzzy matrix. Meenakshi A.R and cokilavany R. [7] introduced the concept of fuzzy 2 normed linear spaces. Nagoorgani A and Kalyani G [5] introduced the fuzzy matrix m – ordering. Zhou Min na [9] introduced the characterizations of the minus ordering in fuzzy matrix set. In this paper we introduce fuzzy adjoint ordering with fuzzy soft matrices using the structure of $M_n(FS)$ the set of $(n \times n)$ fuzzy adjoint ordering with fuzzy soft matrices is introduce and its applied adjoint of fuzzy soft matrix in decision making problem

II. Priliminaries

In this section, we recall some basic essential notion of fuzzy soft set theory and fuzzy soft matrices

Soft set 2.1 : Let U be an initial set and E be a set of parameters. Let $P(U)$ denotes the power set of U . let $A \subseteq E$. A pair (F_A, E) is called a soft set over U , where F_A is a mapping given by $F_A: E \rightarrow P(U)$. such that $F_A(e) = \Phi$ if $e \notin A$. Here F_A is called approximate fraction of the soft set (F_A, E) . The set $F_A(e)$ is called e – approximate value set which consist of related objects of the parameter $e \in E$. In other words, a soft set over U is a parameterized family of subsets of the universe U .

Example 2.1 : Let $U = \{u_1, u_2, u_3, u_4\}$ be a set of four shirts and $E = \{\text{blue } (e_1), \text{ green } (e_2), \text{ yellow } (e_3)\}$ be a set of parameters if $A = \{e_1, e_3\} \subseteq E$. Let $F_A(e_1) = \{u_1, u_3, u_4\}$ and $F_A(e_3) = \{u_2, u_2, u_4\}$ then we write the soft set $(F_A, E) = \{(e_1, \{u_1, u_3, u_4\}), (e_3, \{u_1, u_2, u_4\})\}$ over U which describe the “Colour of the shirts” which Mr. X is going to buy.

We may represent the soft set in the following form.

U	Blue(e1)	Green(e2)	Yellow(e3)
U ₁	1	0	1
U ₂	0	0	1
U ₃	1	0	0
U ₄	1	0	1

Table : 2.1.1

FUZZY SOFT SET2.2: Let U be an initial universe set and E be a set of parameters. Let P(U) denotes the set of all fuzzy sets of U. Let A ⊆ E. A pair (F_A, E) is called a fuzzy soft set (FSS) over U, where F_A is a mapping given by F_A : E → P(U) such that F_A e = Φ if e ∉ A, Φ is a null fuzzy set.

Example 2.2 : Consider the example 2.1 here we cannot express with only two real numbers 0 and 1, we can characterized it by a memberships function instead of crisp number 0 and 1, which associate with each element a real number in the interval [0,1] then.

(F_A, E) = { F_A(e1) = {(u1,0.3), (u2,0.9), (u3,0.4), (u4,0.7)},
 F_A(e3) = {(u1,0.5), (u2,0.6), (u4,0.8)} } is the fuzzy soft set representing the “Colour of the Shirts”

which Mr-X is going to buy. We may represent the fuzzy soft set in the following form.

U	Blue(e1)	Green(e2)	Yellow(e3)
U ₁	0.3	0.0	0.5
U ₂	0.9	0.0	0.6
U ₃	0.4	0.0	0.0
U ₄	0.7	0.0	0.8

Table 2.2

2.3 The complement of a fuzzy soft set : The complement of a fuzzy soft set (F,A) is denoted by (F,A)^c and is defined by

(F,A)^c = (F^c, A) where, F^c : A → P(U) is mapping given by F^c(e) = [F(e)]^c ∀ e ∈ A.

2.4 Fuzzy Soft Matrices (FSM) : Let (F_A, E) be a fuzzy soft set over U, then a subset of U X E is uniquely defined by R_A = {(u,e); e ∈ A, u ∈ F_A(e)} which is called relation form of (F_A, E) the characteristic function of R_A is written by μ_{RA} : U X E → [0,1], where μ_{RA}(u,e) ∈ [0,1] is the membership value of u ∈ E for each e ∈ U. If [μ_{ij}] = μ_{RA}(u_i, e_j), we can define a matrix

$$[\mu_{ij}]_{m \times n} = \begin{bmatrix} \mu_{11} & \mu_{12} & \dots & \mu_{1n} \\ \mu_{21} & \mu_{22} & \dots & \mu_{2n} \\ \dots & \dots & \dots & \dots \\ \mu_{m1} & \mu_{m2} & \dots & \mu_{mn} \end{bmatrix}$$

which is called an mxn soft matrix of the soft set (F_A, E) over U.

Therefore we can say that a fuzzy soft set (F_A, E) is uniquely characterized by the matrix $[\mu_{ij}]_{m \times n}$ and both concepts are interchangeable.

The set $m \times n$ fuzzy soft matrices over U will be denoted by $FSM_{m \times n}$.

Example 2.3

Assume that $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a universal set and $E = \{e_1, e_2, e_3, e_4\}$ is a set of all parameters.

If $A \subseteq E = \{e_1, e_2, e_3, e_4\}$ and $(F_1, E) = \{ F(e_1) = \{(u_1, 0.3), (u_2, 0.4), (u_3, 0.6), (u_4, 0.7), (u_5, 0.5)\}$

$F(e_2) = \{(u_1, 0.2), (u_2, 0.7), (u_3, 0.1), (u_4, 0.8), (u_5, 0.6)\}$

$F(e_3) = \{(u_1, 0.1), (u_2, 0.3), (u_3, 0.5), (u_4, 0.4), (u_5, 0.9)\}$

Then the fuzzy soft matrix $[\mu_{ij}]$ can be written as

$$[\mu_{ij}]_{m \times n} = \begin{bmatrix} 0.3 & 0.2 & 0.0 & 0.1 \\ 0.4 & 0.7 & 0.0 & 0.3 \\ 0.6 & 0.1 & 0.0 & 0.5 \\ 0.7 & 0.8 & 0.0 & 0.4 \\ 0.5 & 0.6 & 0.0 & 0.9 \end{bmatrix}$$

Definition 2.5 [15]: Let $U = \{c_1, c_2, \dots, c_m\}$ be the universal set and E be the set of parameters given by $E = \{e_1, e_2, e_3, \dots, e_n\}$. Let the set of all $m \times n$ fuzzy soft matrices over U be $FSM_{m \times n}$. Let $A, B \in FSM_{m \times n}$. Where $A = [a_{ij}]_{m \times n}$, $a_{ij} = (\mu_{j_1}c(i), \mu_{j_2}c(i))$ and $B = [b_{ij}]_{m \times n}$, $b_{ij} = (\lambda_{j_1}c(i), \lambda_{j_2}c(i))$. To avoid degenerate case we assume that $\min(\mu_{j_1}c(i), \lambda_{j_1}c(i)) \geq \max(\mu_{j_2}c(i), \lambda_{j_2}c(i))$ for all i and j . we define the operation “addition(+)” between, $A+B=C$, where $C = [c_{ij}]_{m \times n}$, $c_{ij} = (\max(\mu_{j_1}c(i), \lambda_{j_1}c(i)), \min(\mu_{j_2}c(i), \lambda_{j_2}c(i)))$. If $\mu_{j_2}c(i) = \lambda_{j_2}c(i) = 0 \forall i, j$, then one definition reduce to $A+B=C$, where $C = [c_{ij}]_{m \times n}$.

$C_{ij} = (\max(\mu_{j_1}c(i), \lambda_{j_1}c(i)), \min(0, 0) (\max(\mu_{j_1}c(i), \lambda_{j_1}c(i)), 0) = \max(\mu_{j_1}c(i), \lambda_{j_1}c(i))$, which is the define of addition(max) of two fuzzy matrice [2] in the usual sense three fuzzy reference is 0.

Definition : 2.6 [15] : Let $A, B \in FSM_{m \times n}$. Let the corresponding membership value, matrices be $M_v(A) = [\{\rho(A)_{ij}\}]_{m \times n}$ and $M_v(B) = [\{\rho(B)_{ij}\}]_{m \times n}$, $i=1,2,3, \dots, m$; then $j=1,2,3, \dots, n$. Then the score matrix $\rho(A, B)$ would be defined as $\rho(A, B) = [\rho_{ij}]_{m \times n}$ where $\rho_{ij} = \rho(A)_{ij} - \rho(B)_{ij}$.

Definition: 2.7 : Let $A = [\mu_{ij}]_{m \times n} \in FSM_{m \times n}$ the determinant of A , denoted as $\det(A)$ or $|A|$, is defined as

$$|A| = \sum_{\pi \in S_n} \mu_1 \pi(1) \mu_2 \pi(2) \dots \mu_n \pi(n)$$

Where the summation is taken over all π of S_n .

Definition: 2.8: The determinant of the fuzzy soft matrix A of order 2 is denoted by

$$|A| = \begin{vmatrix} (\mu_{11}c(1), \mu_{12}c(1)) & (\mu_{21}c(1), \mu_{22}c(1)) \\ (\mu_{11}c(2), \mu_{12}c(2)) & (\mu_{21}c(2), \mu_{22}c(2)) \end{vmatrix}$$

$$= [\max \{ \min(\mu_{11}c(1), \mu_{21}c(2)) \}, \min \{ \max(\mu_{21}c(1), \mu_{22}c(2)), \max(\mu_{22}c(1), \mu_{12}c(2)) \}].$$

Definition: 2.9 : The determinant of the fuzzy soft matrix B order 3 is given by

$$|B| = \begin{vmatrix} (\lambda_{11}c(1), \lambda_{12}c(1)) & (\lambda_{21}c(1), \lambda_{22}c(1)) & (\lambda_{31}c(1), \lambda_{32}c(1)) \\ (\lambda_{11}c(2), \lambda_{12}c(2)) & (\lambda_{21}c(2), \lambda_{22}c(2)) & (\lambda_{31}c(2), \lambda_{32}c(2)) \\ (\lambda_{11}c(3), \lambda_{12}c(3)) & (\lambda_{21}c(3), \lambda_{22}c(3)) & (\lambda_{31}c(3), \lambda_{32}c(3)) \end{vmatrix}$$

$$|B| = (\lambda_{11}c(1), \lambda_{12}c(1)) \begin{vmatrix} (\lambda_{21}c(2), \lambda_{22}c(2)) & (\lambda_{31}c(2), \lambda_{32}c(2)) \\ (\lambda_{21}c(3), \lambda_{22}c(3)) & (\lambda_{31}c(3), \lambda_{32}c(3)) \end{vmatrix}$$

$$+ (\lambda_{21}c(1), \lambda_{22}c(1)) \begin{vmatrix} (\lambda_{11}c(2), \lambda_{12}c(2)) & (\lambda_{31}c(3), \lambda_{32}c(2)) \\ (\lambda_{11}c(3), \lambda_{12}c(3)) & (\lambda_{31}c(3), \lambda_{32}c(3)) \end{vmatrix}$$

$$+ (\lambda_{31}c(1), \lambda_{32}c(1)) \begin{vmatrix} (\lambda_{11}c(2), \lambda_{12}c(2)) & (\lambda_{21}c(2), \lambda_{22}c(2)) \\ (\lambda_{11}c(3), \lambda_{12}c(3)) & (\lambda_{21}c(3), \lambda_{22}c(3)) \end{vmatrix}$$

$$= (\lambda_{11}c(1), \lambda_{12}c(1)) [\max \{ \min((\lambda_{21}c(2), \lambda_{31}c(3)), \min((\lambda_{31}c(2), \lambda_{21}c(3))), \min \{ \max(\lambda_{22}c(2), \lambda_{32}c(3)), \max(\lambda_{32}c(2), \lambda_{22}c(3)) \}]$$

$$+ (\lambda_{21}c(1), \lambda_{22}c(1)) [\max \{ \min((\lambda_{11}c(2), \lambda_{31}c(3)), \min((\lambda_{31}c(2), \lambda_{11}c(3))), \min \{ \max(\lambda_{12}c(2), \lambda_{32}c(3)), \max(\lambda_{32}c(2), \lambda_{12}c(3)) \}]$$

$$+(\lambda_{31}c(1), \lambda_{32}c(1)) [\max\{\min((\lambda_{11}c(2), \lambda_{21}c(3)), \min((\lambda_{21}c(3), \lambda_{11}c(3))), \min\{\max(\lambda_{12}(2), \lambda_{22}c(3)), \max(\lambda_{22}c(2), \lambda_{13}c(3))\}\}]$$

III. Adjoint Of A Fuzzy Soft Matrices

In this section, we introduce adjoint of a fuzzy soft matrix.

Definition: 3.1 : The adjoint matrix of an nxn fuzzy soft matrix. A is denoted by adj A and is defined as $b_{ij} = |A_{ij}|$ where $|A_{ij}|$ is the determinant of the (n-1) x (n-1) fuzzy soft matrix formed by deleting row j and column i from A and $B = \text{adj } A$

Definition: 3.2

The adjoint of the fuzzy soft matrix A in the order of 3 to be defined in our way.

$$b_{11} = |A_{11}| = \begin{vmatrix} (\mu_{21}(c_2), \mu_{22}(c_2) \mu_{31}(c_2), \mu_{32}(c_2)) \\ (\mu_{21}(c_3), \mu_{22}(c_3) \mu_{31}(c_3), \mu_{32}(c_3)) \end{vmatrix}$$

$$= [\text{Max}\{\min(\mu_{21}(c_2), \mu_{31}(c_3) \min(\mu_{31}(c_2), \mu_{21}(c_3))\}, \text{Min}\{\max(\mu_{22}(c_2), \mu_{31}(c_3) \max(\mu_{32}(c_2), \mu_{22}(c_3))\}]$$

$$b_{12} = |A_{12}| = \begin{vmatrix} (\mu_{11}(c_2), \mu_{12}(c_2) \mu_{31}(c_2), \mu_{32}(c_2)) \\ (\mu_{11}(c_3), \mu_{12}(c_3) \mu_{31}(c_3), \mu_{32}(c_3)) \end{vmatrix}$$

$$= [\text{Max}\{\min(\mu_{11}(c_1), \mu_{21}(c_2) \min(\mu_{21}(c_1), \mu_{11}(c_2))\}, \text{Min}\{\max(\mu_{12}(c_1), \mu_{22}(c_2) \max(\mu_{22}(c_1), \mu_{12}(c_2))\}].$$

Similarly we find to the others.

Then

$$\text{adj } A = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \text{ or } \begin{pmatrix} |A_{13}| & |A_{23}| & |A_{33}| \\ |A_{21}| & |A_{31}| & |A_{32}| \\ |A_{11}| & |A_{12}| & |A_{13}| \end{pmatrix}$$

Example: 3.1

Let as consider example 3.1

$$A = \begin{bmatrix} (0.3,0.0) & (0.7,0.1) & (0.6,0.2) \\ (0.5,0.1) & (0.9,0.5) & (0.7,0.0) \\ (0.6,0.3) & (0.7,0.1) & (0.7,0.2) \end{bmatrix}$$

Here

$$b_{11} = |A_{11}| = \begin{vmatrix} (0.9,0.5) & (0.7,0.0) \\ (0.7,0.1) & (0.7,0.2) \end{vmatrix}$$

$$= [\max\{\min(0.9,0.7), \min(0.7,0.7)\}, \min\{\max(0.0,0.1), \max(0.5,0.2)\}].$$

$$= [\max(0.7,0.7), \min(0.1,0.5)]$$

$$b_{11} = (0.7,0.1)$$

Similarly

$$b_{12} = |A_{12}| = (0.6, 0.2)$$

$$b_{13} = |A_{13}| = (0.6, 0.1)$$

$$b_{21} = |A_{21}| = (0.7, 0.2)$$

$$b_{22} = |A_{22}| = (0.6, 0.0)$$

$$b_{23} = |A_{23}| = (0.6, 0.1)$$

$$b_{31} = |A_{31}| = (0.7, 0.1)$$

$$b_{32} = |A_{32}| = (0.5, 0.0)$$

$$b_{33} = |A_{33}| = (0.5, 0.1)$$

Hence the adjoint of the matrix A will be come.

$$\text{adj } A = \begin{bmatrix} (0.7,0.1) & (0.6,0.2) & (0.6,0.1) \\ (0.7,0.2) & (0.5,0.0) & (0.6,0.0) \\ (0.6,0.1) & (0.7,0.1) & (0.5,0.1) \end{bmatrix}$$

Proposition: 3.1

For A and $B \in \text{FSM}_{n \times n}$ we have the following.

1. $A \leq B \Rightarrow \text{adj } A \leq \text{adj } B$.
2. $\text{adj } A + \text{adj } B \leq \text{adj } (A+B)$.
3. $\text{adj } A^T = (\text{adj } A)^T$

Proof:

1. Let $C = \text{adj } A$ and $D = \text{adj } B$ then be $C_{ij} = \sum_{\pi \in S_{n-1j}} \prod_{t \in \pi} \mu_t \prod (t)$ and $d_{ij} = \sum_{\pi \in S_{n-1j}} \prod_{t \in \pi} \mu_t \chi(t)$ it is clear that $C_{ij} \leq d_{ij}$ because $\mu_t \prod (t) \leq \chi(t) \prod (t)$ for every $t \in \pi$.
2. Since $A, B \leq A + B$, it is clear that $\text{adj } A, \text{adj } B \leq \text{adj } (A + B)$ and so $\text{adj } A + \text{adj } B \leq \text{adj } (A + B)$.
3. Let $B = \text{adj } A$ and $C = \text{adj } A^T$

Then $b_{ij} = \sum_{\pi \in S_{n-1j}} \prod_{t \in \pi} \mu_t \prod (t)$ and $C_{ij} = \sum_{\pi \in S_{n-1j}} \prod_{t \in \pi} \mu_t \prod (t)$.

Which is the element $b_{ij} = \text{Hence } (\text{adj } A)^T = \text{adj } A^T$.

Proposition: 3.2

For $A \in \text{FSM}_{m \times n}$ we have $|A| = |\text{adj } A|$.

Proof:

By Definition 3.1

$$\text{adj } A = \begin{bmatrix} |A_{11}| & |A_{12}| & \dots & |A_{1n}| \\ |A_{21}| & |A_{22}| & \dots & |A_{2n}| \\ \vdots & \vdots & \dots & \vdots \\ |A_{n1}| & |A_{n2}| & \dots & |A_{nn}| \end{bmatrix}$$

$$\begin{aligned} |\text{adj } A| &= \sum_{\pi \in S_n} |A_{1\pi(1)}| |A_{2\pi(2)}| \dots |A_{n\pi(n)}| \\ &= \sum_{\pi \in S_n} \prod_{i=1}^n |A_{i\pi(i)}| \\ &= \sum_{\pi \in S_n} \left[\prod_{i=1}^n \left(\sum_{\theta \in S_{n-1\pi(i)}} \prod_{t \in \theta} \pi_t \theta(t) \right) \right] \\ &= \sum_{\pi \in S_n} \left[\left(\sum_{\theta \in S_{n-1\pi(1)}} \prod_{t \in \theta} \pi_t \theta(t) \right) \left(\sum_{\theta \in S_{n-2\pi(2)}} \prod_{t \in \theta} \pi_t \theta(t) \right) \dots \left(\sum_{\theta \in S_{n-1\pi(n)}} \prod_{t \in \theta} \pi_t \theta(t) \right) \right] \\ &= \sum_{\pi \in S_n} \left[\left(\prod_{t \in \pi(1)} \pi_t \theta_1(t) \right) \left(\prod_{t \in \pi(2)} \pi_t \theta_2(t) \right) \dots \left(\prod_{t \in \pi(n)} \pi_t \theta_n(t) \right) \right] \\ &= \sum_{\pi \in S_n} \left[\left(\pi_2 \theta_1(2) \pi_3 \theta_1(3) \dots \pi_n \theta_1(n) \right) \left(\pi_1 \theta_2(1) \pi_3 \theta_2(3) \dots \pi_n \theta_2(n) \right) \dots \right. \\ &\quad \left. \left(\pi_1 \theta_n(1) \pi_2 \theta_n(2) \dots \pi_{n-1} \theta_n(n-1) \right) \right] \\ &= \sum_{\pi \in S_n} \left[\left(\pi_1 \theta_2(1) \pi_1 \theta_3(1) \dots \pi_1 \theta_n(1) \right) \left(\pi_2 \theta_1(2) \pi_2 \theta_3(2) \dots \pi_2 \theta_n(2) \right) \right. \\ &\quad \left. X \left(\pi_3 \theta_1(3) \pi_3 \theta_2(3) \pi_3 \theta_4(3) \dots \pi_3 \theta_n(3) \right) \dots X \left(\pi_n \theta_1(n) \pi_n \theta_2(n) \dots \pi_n \theta_{n-1}(n) \right) \right] \\ &= \sum_{\pi \in S_n} \left[\left(\pi_1 \theta_{f_1}(1) \pi_2 \theta_{f_2}(2) \dots \pi_n \theta_{f_n}(n) \right) \right]. \end{aligned}$$

For some $f_h \in \{1, 2, \dots, n\} \setminus \{h\}$, $h = 1, 2, \dots, n$ However because $\pi_{h\theta_h}(h) \neq \pi_{h\theta_h}(f_h)$, We can see that $\pi_{h\theta_h}(n) = \pi_{h\theta_h}(f_h)$. Therefore $|\text{adj } A| = \sum_{\pi \in S_n} \pi_{1\pi(1)} \pi_{2\pi(2)} \dots \pi_{n\pi(n)}$ which is the expansion of $|A|$.

Hence the proof.

IV. Application Of Fsm In Decision Making

In this section, we are submitting the problem which is based on determinant and adjoint of FSM.

4.1 Determinant and adjoint of Fuzzy soft Matrices in Decision Making.

In this field, let us suppose U is a set of certain number of cities, E is a set of parameters related to healthy environment of a city which vary with time. We construct a fuzzy soft set (F, E) over U representing the healthy environment of a cities at an instant t , where F is a mapping $F: E \rightarrow \mathcal{F}(U)$, $\mathcal{F}(U)$ is the set of all fuzzy subsets of U . we further construct another, fuzzy soft set (G, E) over U representing the healthy environment of the cities at an instant t^1 . The matrices A and B corresponding to the fuzzy soft set (F, E) and (G, E) are constructed. We compute the complements $(F, E)^c$ and $(G, E)^c$ and write the matrices A and B corresponding to $(F, E)^c$ and $(G, E)^c$ respectively.

V. Algorithm

1. Input the fuzzy soft matrices (F, E) and (G, E). Also write the fuzzy soft matrices A and B corresponding to (F, E) and (G, E) respectively.
2. Write the fuzzy soft matrices (F, E)^C and (G, E)^C. Also write the fuzzy soft matrices A and B corresponding to (F, E)^C and (G, E)^C respectively.
3. Compute adj (A+B) and MV(adj A+B).
4. Compute adj ($\bar{A} + \bar{B}$) and MV(adj $\bar{A} + \bar{B}$).
5. Compute the score matrix $S_{(adj A+B) adj A+B}$.
6. Compute the total score S_i for each C_i in U.
7. Find $SK = \max (S_i)$. Then we conclude that the city CK has the maximum healthy environment between the line instances t and t¹ respectively.
8. If SK has more than one value, then go to step(1) and repeat the process by reassessing the parameters for healthy environment.

VII. Case Study

Let (F, E) and (G, E) be two fuzzy soft set representing the healthy environment of four cities U= {C1, C2, C3}. At instants t and t¹ respectively. Let E = {e1 (less – crowdness), e2 (Noise - free), e3 (non - pollution) } be the set of parameters which would vary with time.

$$(F, E) = \{F(e_1) = \{(C_1, 0.7, 0), (C_2, 0.5, 0), (C_3, 0.6, 0)\},$$

$$F(e_2) = \{(C_1, 0.9, 0), (C_2, 0.6, 0), (C_3, 0.7, 0)\},$$

$$F(e_3) = \{(C_1, 0.5, 0), (C_2, 0.4, 0), (C_3, 0.9, 0)\},$$

$$(G, E) = \{G(e_1) = \{(C_1, 0.6, 0), (C_2, 0.4, 0), (C_3, 0.5, 0)\},$$

$$G(e_2) = \{(C_1, 0.8, 0), (C_2, 0.9, 0), (C_3, 0.3, 0)\},$$

$$G(e_3) = \{(C_1, 0.5, 0), (C_2, 0.9, 0), (C_3, 0.6, 0)\},$$

There two fuzzy soft sets are represented by the following fuzzy soft matrices respectively,

$$A = \begin{matrix} e_1 & e_2e_3 & e_1 & e_2e_3 \\ C1 & \begin{bmatrix} (0.7,0) & (0.9,0) & (0.5,0) \end{bmatrix} \\ C2 & \begin{bmatrix} (0.5,0) & (0.6,0) & (0.4,0) \end{bmatrix} \\ C3 & \begin{bmatrix} (0.6,0) & (0.7,0) & (0.9,0) \end{bmatrix} \end{matrix} \quad \text{and} \quad B = \begin{matrix} C1 & \begin{bmatrix} (0.6,0) & (0.8,0) & (0.5,0) \\ (0.4,0) & (0.9,0) & (0.9,0) \\ (0.5,0) & (0.3,0) & (0.6,0) \end{bmatrix} \end{matrix}$$

The fuzzy soft matrices are representing the healthy environment of the three cities U= {C1, C2, C3} at instants t and t¹ respectively all given by.

$$\bar{A} = \begin{matrix} e_1 & e_2e_3 & e_1 & e_2e_3 \\ C1 & \begin{bmatrix} (1.0,7) & (1.0,9) & (1.0,5) \end{bmatrix} \\ C2 & \begin{bmatrix} (1.0,5) & (1.0,6) & (1.0,4) \end{bmatrix} \\ C3 & \begin{bmatrix} (1.0,6) & (1.0,7) & (1.0,9) \end{bmatrix} \end{matrix} \quad \text{and} \quad \bar{B} = \begin{matrix} C1 & \begin{bmatrix} (1.0,6) & (1.0,8) & (1.0,5) \\ (1.0,4) & (1.0,9) & (1.0,9) \\ (1.0,5) & (1.0,3) & (1.0,6) \end{bmatrix} \end{matrix}$$

Then the fuzzy soft matrix adj (A+ B) represents the maximum membership for of healthy environment of the cities between the time instances t and t¹.

$$A + B = \begin{matrix} e_1 & e_2e_3 \\ C1 & \begin{bmatrix} (0.7,0) & (0.9,0) & (0.5,0) \\ (0.5,0) & (0.9,0) & (0.9,0) \\ (0.6,0) & (0.7,0) & (0.9,0) \end{bmatrix} \end{matrix}$$

$$\text{Then adj (A+ B)} = \begin{matrix} e_1 & e_2e_3 \\ C1 & \begin{bmatrix} (0.9,0) & (0.6,0) & (0.6,0) \\ (0.9,0) & (0.7,0) & (0.7,0) \\ (0.9,0) & (0.7,0) & (0.7,0) \end{bmatrix} \end{matrix}$$

$$MV(\text{adj A+ B}) = \begin{matrix} e_1e_2e_3 \\ C1 & \begin{bmatrix} (0.9) & (0.6) & (0.6) \\ (0.9) & (0.7) & (0.7) \\ (0.9) & (0.7) & (0.7) \end{bmatrix} \end{matrix}$$

Again the fuzzy soft matrix adj (A + B) represent the maximum membership function of non – healthy environment of the cities between the time instances t and t¹.

$$\bar{A} + \bar{B} = \begin{matrix} e_1 & e_2e_3 \\ C1 & \begin{bmatrix} (1.0,4) & (1.0,5) & (1.0,4) \\ (1.0,5) & (1.0,5) & (1.0,6) \\ (1.0,6) & (1.0,5) & (1.0,6) \end{bmatrix} \end{matrix}$$

$$e_1 e_2 e_3$$

$$\text{adj}(\bar{A} + \bar{B}) = \begin{matrix} C1 \\ C2 \\ C3 \end{matrix} \begin{bmatrix} (1,0.6) & (1,0.8) & (1,0.5) \\ (1,0.4) & (1,0.6) & (1,0.4) \\ (1,0.5) & (1,0.3) & (1,0.6) \end{bmatrix}$$

$$e_1 e_2 e_3$$

$$MV(\text{adj}(\bar{A} + \bar{B})) = \begin{matrix} C1 \\ C2 \\ C3 \end{matrix} \begin{bmatrix} 0.6 & 0.5 & 0.6 \\ 0.5 & 0.5 & 0.4 \\ 0.4 & 0.5 & 0.4 \end{bmatrix}$$

We now calculate the some matrix $S_{(\text{adj } A+B) \text{ adj } \bar{A} + \bar{B})}$ and total score for healthy environment of each city.

$$e_1 \quad e_2 e_3$$

$$S_{(\text{adj } A+B) \text{ adj } (\bar{A} + \bar{B})} = \begin{matrix} C1 \\ C2 \\ C3 \end{matrix} \begin{bmatrix} 0.3 & 0.1 & 0 \\ 0.4 & 0.2 & 0.3 \\ 0.5 & 0.2 & 0.3 \end{bmatrix}$$

$$\text{Total score for healthy environment } \begin{matrix} S_1 \\ S_2 \\ S_3 \end{matrix} \begin{vmatrix} 0.4 \\ 0.9 \\ 1.0 \end{vmatrix}$$

We see that S_3 has the maximum value and thus conclude that the city C_3 has got highest total score and hence the city having the most healthy environment among all the cities between the instances t and t^1 .

VIII. Conclusion

We have applied the adjoint of fuzzy soft matrix in decision making problem. We hope that give approach will be useful to handle different uncertain problem.

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