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Basics on Chromatic Polynomials in Graphs

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Abstract
This article is a general introduction of chromatic polynomials.In this,Chromatic polynomials are defined,their connection between the theory of chromatic polynomials and coloring of graphs.Also it explains the chromatic polynomials of total graphs .It gives the basic concepts of chromatic polynomials in graph theory.

Keywords: *Chromatic polynomial; coloring of graph ,Total graph,bond lattice*

I. Introduction:

Kirchhoff [1] introduced the chromatic polynomial in 1912 as an attempt to prove the four colour theorem. Today we usually define the chromatic polynomial for arbitrary graphs extended by Whithey [2]. After a while Reed [3] studied about the chromatic polynomials. In addition to that Yap [4] added his contribution on total colourings of graphs. As well Meredith [5] discussed about coefficients of chromatic polynomials in his learning. Further Erdos and Wilson [6] put in on the chromatic indexes.

II. Basics of Graph theory

Definitions[7]:

2.1 A graph G consists of a non-empty finite set V(G) (called vertex set)of elements called vertices,and a finite set E(G) (called edge set) of unordered pair of elements of V(G) called edges.

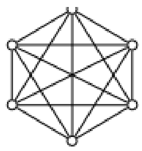
2.2 Two vertices a and b of a graph G are said to be adjacent if there is an edge ab joing them.

2.3 The degree of a vertex v of a graph G is the number of edges incident with v.

2.3 A simple graph is one in which there is atmost one edge joining a given pair of vertices and there are no loops,or edge joining a given vertex with itself.

2.4 Complete graph:

The graph $G = K_V$ is the **complete graph** on V, if every two vertices are adjacent: $E = E(V)$. All complete graphs of order n are isomorphic with each other, and they will be denoted by K_n.



2.5 A connected graph in which the degree of each vertex is 2 is a cycle graph,a cycle graph with n vertices is denoted by Cn

2.6 A path graph on n vertices is the graph obtained when an edge is removed from the cycle graphCn.A path graph of n vertices is denoted by Pn.

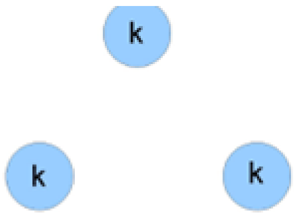
2.7 Graph coloring:A coloring of a graph G so that adjacent vertices are different colors is called a proper coloring of the graph.

2.8 A graph G is k-colorable if we can assign one of k colors to each vertex to achieve a proper coloring.

2.9 A graph G is k-chromatic or has chromatic number k if G is k-colorable but not k-1 colorable.Symbolically,let x be a function such that x(G) =k,where k is chromatic number of G. We denote if x(G) =k,then G is n-colorable for n≥k

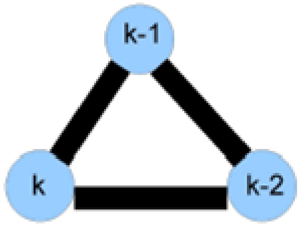
2.10 Chromatic polynomials: If G is a simple graph, $P_G(k)$ is the number of ways we can achieve a proper coloring on the vertices of G with k colors and P_G is called theChromatic function of G .
If K<x(G) then $P_G(k)=0$

2.11 Examples:



Calculating the chromatic function of the null graph N3

Calculating the chromatic function of the path graph P3



Calculating the chromatic function of the K3

For the complete graph K3, we start by selecting a random vertex and it can be colored k ways.If we move from this vertex to any other,we notice this second one can be colored k-1 ways as it is adjacent to the first.The third and the final vertex can be colored k-2 ways as it is adjacent to both of the first two.We observe that K3 can be colored k (k-1)(k-2) ways with k colors

III. Total Graphs

3.1. Definition [8]: Let G be any graph .The total graph of G, T (G) is that a graph whose vertex set is V (G) E (G) and in which two vertices are adjacent if and only if they are adjacent are incident in G.

3.2. Example: Fig. 1 represents a (4, 3)-connected graph G and its total graph T(G).

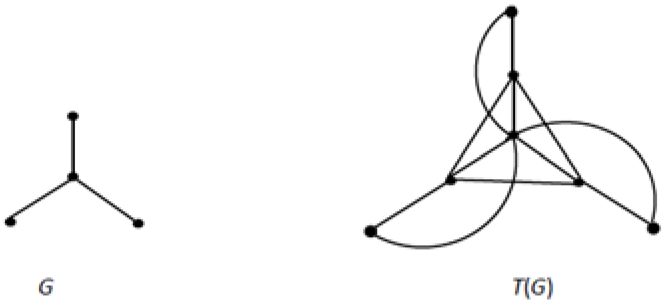


Fig 1 A graph G with its total graph T(G)

3.3. Definition [4]: A partially order set with the property that every pair of elements has a greatest lower bound and least upper bound is called a lattice.
G T(G)

3.4. Example: Let D12= {1,2,3,4,6,12} be the set of all divisors of 12 under the relation divides is a lattice .

3.5. Definition [9]: A partition of a set S is defined to be subsets of S which are disjoint and whose union is S .Each element of a partition is known as a part.

3.6. Example: Let S = {1,2,3,4}.Two different partitions of S which are labeled as P and Q , let P={{1,2},{3},{4}} and Q = {{1,2,3},{4}}.Two different parts of P are

{1,2},{4}.

3.7 A bond of a graph G is a partition of its vertices such that all vertices in the same part are connected within the graph.That is they are adjacent or there exists a path between them in the graph that includes only other vertices in the same part. The set of bonds of a graph form the bond lattice.

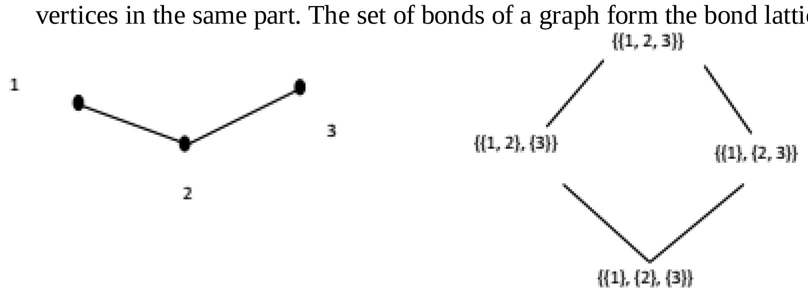


Fig 2 A graph and its bond lattice

IV. Conclusion:

In this paper we studied the general introduction of chromatic polynomials.In this paper Chromatic polynomials are defined,their connection between the theory of chromatic polynomials and coloring problems.Also it explains the chromatic polynomials of total graphs.It gives the basic concepts of chromatic polynomials in graph theory.

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