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A NEW PROPERTIES OF ALGEBRIC STRUCTURE ON HEXAGONAL FUZZY NUMBER

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ABSTRACT
The objective of this paper is to introduce an algebraic structure on ($\mathcal{H}$) the family hexagonal fuzzy number (HFN) is namely group then verify all the necessary property of group for the family of HFN and then we write the basic theorem based on this.*
KEY WORDS: *Fuzzy number, Hexagonal Fuzzy Number, Algebric Group.*

INTRODUCTION

Fuzzy setswere introduced by Zadeh in 1965 to represent the information possessing non-statistical certainties. Different types of fuzzy set are defined in order to clear the vagueness of the assisting problems. A **Fuzzy number**is quantity whose values are imprecise, then exact as in the case with single- valued function. the usual arithmetic operations on real numbers can be extended to the ones defined on fuzzy numbers by means of Zadeh's Extension principle and then some of the note working contributions on fuzzy number. Mizumotoand Tanoak internal arithmetic was first suggested by Dwyer. The various operations on fuzzy numbers were also available in the literature. Due to error in measuring technique, instrumental faultiness etc., some data in our observation cannot be accurately determined. This leads to introduce a new type of fuzzy number called the Hexagonal Fuzzy Number (HFN). Generally, a hexagonal fuzzy number is a 6-tuple subset of areal number R having six parameters. We introduce HFS is a group and under the addition,multiplication.then based theorem's as follows that algebraic structure of the hexagonal fuzzy number.

In this paper is framed as follows: Section 2 discusses the basic definitions andconcepts of hexagonal fuzzy numbers is reviewed. In section 3 algebraic structure on hexagonal fuzzy numbers and group for the family of HFN basic theorems. In section 4concludes the paper.

PRLIMINARIES

2.1 FUZZY SET:

Let X be a nonempty set and A fuzzy set $\tilde{F}(x)$is defined as $\tilde{F}$ ={(x, $\mu\tilde{F}(x)$/x∈X} where $\mu\tilde{F}(x)$ is called the membership function which maps each element of X to a value between 0 and 1.

2.2. FUZZY NUMBER :

A fuzzy number is a generalized regular real number which refers to a connected set ofpossible values of weights between 0 and 1. This weight is called the membership function.

A fuzzy number F is a convex normalized fuzzy set on the real line R such that:

- There exist at least one X € R with$\mu\tilde{F}$ (x) =1
- $\mu\tilde{F}$ (x) is piecewise continuous

2.3. HEXAGONALFUZZYNUMBER(HFN):

2.3.1. DEFINITION [9]:

A fuzzy number$\tilde{F}_H$is a hexagonal fuzzy number denoted by $\tilde{F}_H = (f_1, f_2, f_3, f_4, f_5, f_6)$ where f_1, f_2, f_3, f_4, f_5 and f_6 are real numbers and its membership function is given below,

$$\mu\tilde{F}(x) = \begin{cases} \frac{1}{2}\frac{x-f1}{f2-f1}, & \text{for } f1 \le x \le f2 \\ \frac{1}{2}+\frac{1}{2}\frac{x-f2}{f3-f2}, & \text{for } f2 \le x \le f3 \\ 1, & \text{for } f3 \le x \le f4 \\ 1-\frac{1}{2}\frac{x-f4}{f5-f4} & \text{for } f4 \le x \le f5 \\ \frac{1}{2}\frac{f6-x}{f6-f5}, & \text{for } f5 \le x \le f6 \\ 0, & \text{for other wise} \end{cases}$$

2.3.2 DEFINITION[9]:

A fuzzy set $\tilde{F}_H$ is defined as asset of real numbers with its membership function having the following characteristics

- $P_1(u)$ is a bounded left continuous non decreasing function over[0,0.5],
- $Q_1(v)$ is a bounded left continuous non decreasing function over[0.5,w],
- $Q_2(v)$ is a bounded left continuous non increasing function over[w,0.5],
- $P_2(v)$ is a bounded left continuous non increasing function over[0.5,0].

REMARK [9]:

When w=1 the hexagonal fuzzy number is a normal fuzzy number

POSITIVE AND NEGATIVE HFN [1]:

Let $\tilde{F}_H$= (f_1, f_2, f_3, f_4, f_5, f_6) is positive hexagonal fuzzy number if a_i> 0 for i=1,2….,6and it is negative hexagonal fuzzy number if a_i< 0 for i=1,2,..,6.

2.3.3 ARITHMATIC OPERATIONS ON HFN[1]:

Let $\tilde{F}_H$= (f_1, f_2, f_3, f_4, f_5, f_6) , $\tilde{G}_H$= (g_1, g_2, g_3, g_4, g_5, g_6) be two hexagonal

- ✓ **Addition** :$\tilde{F}_H + \tilde{G}_H = (f_1+g_1, f_2+g_2, f_3+g_3, f_4+g_4, f_5+g_5, f_6+g_6)$
- ✓ **Subtraction** :: $\tilde{F}_H - \tilde{G}_H = (f_1-g_6, f_2-g_5, f_3-g_4, f_4-g_3, f_5-g_2, f_6-g_1)$
- ✓ **Multiplication** :$\tilde{F}_H * \tilde{G}_H = (f_1*g_1, f_2*g_2, f_3*g_3, f_4*g_4, f_5*g_5, f_6*g_6)$

2.3.4 EXISTING METHOD [11]:

The magnitude of a HFN $\tilde{F}_H$= (f_1, f_2, f_3, f_4, f_5, f_6) , is defined as

$$R(\tilde{F}_H) = \left(\frac{2f1+3f2+4f3+4f4+3f5+2f6}{18}, \frac{5w}{18}\right)$$

2.3.5 PROPOSED RANKING OF HFNs:

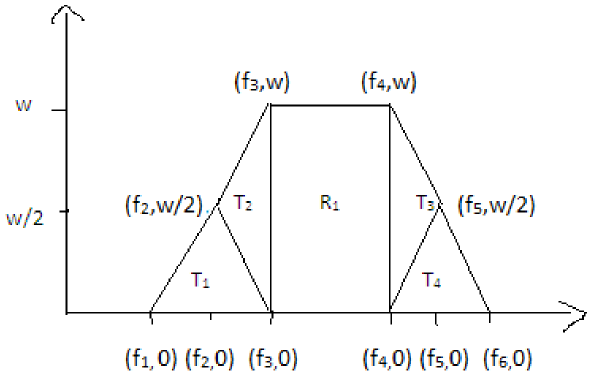

Figure 4.1 Ranking of Hexagonal Fuzzy Number

ALGEBRIC STRUCTURE ON HFNs

3.1 DEFINITION:

A non – empty set Htogether with a binary operation $*:\mathcal{H} * \mathcal{H} \dashrightarrow \mathcal{H}$ *is* called a **group** if the following condition are satisfied.

(1) * is closure

Let $\tilde{F}_H$, $\tilde{G}_H \in \mathcal{H}$

IsThen $\tilde{F}_H * \tilde{G}_H \in \mathcal{H}$

(2) * is associative

Let $\tilde{P}_H$, $\tilde{Q}_H$, $\tilde{R}_H \in \mathcal{H}$ then $\tilde{P}_H * (\tilde{Q}_H * \tilde{R}_H) = (\tilde{P}_H * \tilde{Q}_H) * \tilde{R}_H$

(3) Identity

There exists an element $\tilde{E}_H \in \mathcal{H}$

such that $\tilde{P}_H * \tilde{E}_H = \tilde{E}_H * \tilde{P}_H = \tilde{P}_H$ for all $\tilde{P}_H \in \mathcal{H}$

(4) **Inverse**

For any element $\tilde{P}_H$ in $\mathfrak{H}$ there exists an element $\tilde{P}_H' \in \mathcal{H}$

such that $\tilde{P}_H * \tilde{P}_H' = \tilde{P}_H' * \tilde{P}_H = E$

$\tilde{P}_H'$ is called the **inverse** of $\tilde{P}_H$.

3.1. Theorem:

Prove that $(\mathcal{H}, +)$ is a group

Proof:

$\mathcal{H}$ means script $\mathfrak{H}$ or hexagonal fuzzy number

- **HFNs is closure**

Let $\tilde{F}_H=(f_1, f_2, f_3, f_4, f_5, f_6)\in \mathcal{H}$

$\tilde{G}_{H}=(g_1, g_2, g_3, g_4, g_5, g_6) \in \mathcal{H}$

To prove that $\tilde{F}_H + \tilde{G}_H \in \mathcal{H}$

$\tilde{F}_H + \tilde{G}_H = (f_1, f_2, f_3, f_4, f_5, f_6)+(g_1, g_2, g_3, g_4, g_5, g_6)= (f_1+g_1, f_2+g_2, f_3+g_3, f_4+g_4, f_5+g_5, f_6+g_6) \in \mathcal{H}$

$\mathcal{H}$ **is** satisfied this condition

- **HFNs is associative**

Let $\tilde{P}_H, \tilde{Q}_H, \tilde{R}_H \in \mathcal{H}$

Then $\tilde{P}_H+(\tilde{Q}_H+\tilde{R}_H)=(\tilde{P}_H+\tilde{Q}_H)+\tilde{R}_H$

L.H.S $=(\tilde{Q}_H+\tilde{R}_H)$

$=(q_1, q_2.q_3, q_4, q_5, q_6) +(r_1, r_2, r_3, r_4, r_5, r_6)$

$= (q_1+r_1, q_2+r_2, q_3+r_3, q_4+r_4, q_5+r_5, q_6+r_6)$

$= \tilde{P}_H+(\tilde{Q}_H+\tilde{R}_H)$

$= (p_1, p_2, p_3, p_4, p_5, p_6) + (q_1+r_1, q_2+r_2, q_3+r_3, q_4+r_4, q_5+r_5, q_6+r_6)$

$= (p_1+q_1+r_1, p_2+q_2+r_2, p_3+q_3+r_3, p_4+q_4+r_4, p_5+q_5+r_5, p_6+q_6+r_6)$

R.H. S$=(\tilde{P}_H+\tilde{Q}_H)$

$= (p_1, p_2, p_3, p_4, p_5, p_6) + (q_1, q_2.q_3, q_4, q_5, q_6)$

$= (p_1+q_1, p_2+q_2, p_3+q_3, p_4+q_4, p_5+q_5, p_6+q_6)$

$=(\tilde{P}_H+\tilde{Q}_H)+\tilde{R}_H$

$= (p_1+q_2, p_2+q_2, p_3+q_3, p_4+q_4, p_5+q_5, p_6+q_6)+ (r_1, r_2, r_3, r_4, r_5, r_6)$

$= (p_1+q_1+r_1, p_2+q_2+r_2, p_3+q_3+r_3, p_4+q_4+r_4, p_5+q_5+r_5, p_6+q_6+r_6)$

R.H. S=L.H. S

$\mathcal{H}$ is satisfied this condition

- **HFNs is identity**

Let $\tilde{P}_H= (p_1, p_2, p_3, p_4, p_5, p_6)$

$\tilde{E}_H= (e_1, e_2, e_3, e_4, e_5, e_6)$

To prove **E is identity**

$\tilde{P}_H+\tilde{E}_H=\tilde{P}_H$

$P_1+e_1=p_1$

$e_i=0$ for all i=1, 2…6

$\tilde{E}_H= (0,0,0,0,0,0) \in \mathcal{H}$

the addition identity is zero (0).

- **HFNs is inverse**

$\tilde{P}_H + \tilde{P}_H' = \tilde{P}_H' + \tilde{P}_H = \tilde{E}_H$
Take $\tilde{P}_H + \tilde{P}_H' = \tilde{E}_H$
$(p_1, p_2, p_3, p_4, p_5, p_6)+(p_1',p_2',p_3',p_4',P_5',p_6')= (e_1, e_2, e_3, e_4, e_5, e_6)$
$(p_1+p_1',p_2+p_2',p_3+p_3',p_4+p_4',p_5+p_5',p_6+p_6') =(e_1, e_2, e_3, e_4, e_5, e_6)$

Equating on both side

$p_1+p_1'=e_1$ (addition identity is 0)

$p_1+p_1'=0$
That is $p_1 = -(p_1')$

Similarly, $P_H' = (-p_1, -p_2, -p_3, -p_4, -p_5, -p_6)$

Hence $\mathcal{H}$ is a group under usual addition.

3.2. Theorem:

Prove that ($\mathcal{H}$ +) is a group

Proof:

- **HFNs is closure**

Let $\tilde{F}_H, \tilde{\boldsymbol{G}}_H \in \mathcal{H}$

To proveLet $\tilde{F}_H \cdot \tilde{\boldsymbol{G}}_H \in \boldsymbol{\mathcal{H}}$

$\tilde{F}_H \cdot \tilde{\boldsymbol{G}}_H= (f_1, f_2, f_3, f_4, f_5, f_6).(g_1, g_2, g_3, g_4, g_5, g_6)$

= (f1.g1, f2.g2, f3.g3, f4.g4, f5.g5, f6.g6) $\in\mathcal{H}$

$\mathcal{H}$ is satisfied the condition.

- **HFNs is associative**

Let $\tilde{P}_H, \tilde{Q}_H, \tilde{R}_H \in \mathcal{H}$

Then $\tilde{\boldsymbol{P}}_H.(\tilde{\boldsymbol{Q}}_H.\tilde{\boldsymbol{R}}_H)=(\tilde{\boldsymbol{P}}_H.\tilde{\boldsymbol{Q}}_H).\tilde{\boldsymbol{R}}_H$

L.H.S = $(\tilde{\boldsymbol{Q}}_H.\tilde{\boldsymbol{R}}_H)$

$=(q_1, q_2.q_3, q_4, q_5, q_6).(r_1, r_2, r_3, r_4, r_5, r_6)$

$= (q_1.r_1, q_2.r_2, q_3.r_3, q_4.r_4, q_5.r_5, q_6.r_6)$

$=\tilde{\boldsymbol{P}}_{\mathbf{H}}.(\tilde{\boldsymbol{Q}}_{\mathbf{H}}.\tilde{\boldsymbol{R}}_{\mathbf{H}})$

$=(p_1, p_2, p_3, p_4, p_5, p_6).(q_1.r_1, q_2.r_2, q_3.r_3, q_4.r_4, q_5.r_5, q_6.r_6)$

$= (p_1.q_1.r_1, p_2.q_2.r_2, p_3.q_3.r_3, p_4.q_4.r_4, p_5.q_5.r_5, p_6.q_6.r_6)$

R.H. S=$(\tilde{\boldsymbol{P}}_{\mathbf{H}}.\tilde{\boldsymbol{Q}}_{\mathbf{H}})$

$= (p_1, p_2, p_3, p_4, p_5, p_6). (q_1, q_2.q_3, q_4, q_5, q_6)$

$= (p_1.q_1, p_2.q_2, p_3.q_3, p_4.q_4, p_5.q_5, p_6.q_6)$

$= (\tilde{\boldsymbol{P}}_{\mathbf{H}}.\tilde{\boldsymbol{Q}}_{\mathbf{H}}).\tilde{\boldsymbol{R}}_{\mathbf{H}}$

$= (p_1.q_2, p_2.q_2, p_3.q_3, p_4.q_4, p_5.q_5, p_6.q_6). (r_1, r_2, r_3, r_4, r_5, r_6)$

$= (p_1.q_1.r_1, p_2.q_2.r_2, p_3.q_3.r_3, p_4.q_4.r_4, p_5.q_5.r_5, p_6.q_6.r_6)$

R.H. S=L.H. S

$\mathcal{H}$ is satisfied this condition….

- **Identity**

$\tilde{\boldsymbol{P}}_{\mathrm{H}}= (p_1, p_2, p_3, p_4, p_5, p_6)$
$\tilde{E}_{\mathrm{H}}= (e_1, e_2, e_3, e_4, e_5, e_6)$

$\tilde{P}_{\mathrm{H}}.\tilde{E}_{\mathrm{H}}= \tilde{P}_{\mathrm{H}}$

$p_1. e_1=p_1$

$e_i=1$ for all i= 1, 2…6

$\tilde{E}_{\mathrm{H}}= (1,1,1,1,1,1)$

The multiplication identity is one (1)

- **Inverse**

Take $\tilde{P}_{\mathrm{H}}.\tilde{P}_{\mathrm{H}}'\in \mathcal{H}$

$\tilde{P}_{\mathrm{H}}.\tilde{P}_{\mathrm{H}}' =\tilde{P}_{\mathrm{H}}'.\tilde{P}_{\mathrm{H}}= \tilde{E}_{\mathrm{H}}$

Take$\tilde{P}_{\mathrm{H}} +\tilde{P}_{\mathrm{H}}' = \tilde{E}_{\mathrm{H}}$

$(p_1, p_2, p_3, p_4, p_5, p_6).(p_1',p_2',p_3',p_4',P_5',p_6')= (e_1, e_2, e_3, e_4, e_5, e_6)$
$(p_1.p_1',p_2.p_2',p_3.p_3',p_4.p_4',p_5.p_5',p_6.p_6')= (e_1, e_2, e_3, e_4, e_5, e_6)$

Equating on both side

$p_1.p_1'=e_1$ (addition identity is 1)

$p_1.p_1'=1$

That is $p_1'= (1/p_1)$

Similarly, $P_H' = (1/p_1, 1/p_2, 1/p_3, 1/p_4, 1/p_5, 1/p_6) \in \mathcal{H}$

Hence $\mathcal{H}$ is a group under usual multiplication

Hence proved theorem

3.3. Theorem:

Let $\mathcal{H}$ be a group. Then

1. Identity element of $\mathcal{H}$ is unique
2. For any $P \in \mathcal{H}$, the inverse of P is unique

Proof:

1. Let $\tilde{E}_H$ and $\tilde{E}_H'$ be two identity elements of $\mathcal{H}$

$\tilde{P}_H * \tilde{E}_H = \tilde{E}_H * \tilde{P}_H = \tilde{P}_H$

$\tilde{P}_H * \tilde{E}_H' = \tilde{E}_H' * \tilde{P}_H = \tilde{P}_H$

Above the equation is equal

So, we take $\tilde{P}_H * \tilde{E}_H = \tilde{P}_H * \tilde{E}_H'$

$(\tilde{P}_H' * \tilde{P}_H) * \tilde{E}_H = (\tilde{P}_H * \tilde{P}_H') * \tilde{E}_H'$ $(\tilde{P}_H * \tilde{P}_H' = 1)$

$1 * \tilde{E}_H = 1 * \tilde{E}_H'$

$\tilde{E}_H = \tilde{E}_H'$

So, identity element is unique

2. Let P′ and P″ be two inverses of P

$\tilde{P}_H.\tilde{P}_H' = \tilde{P}_H'.\tilde{P}_H = \tilde{E}_H$

$\tilde{P}_H.\tilde{P}_H'' = \tilde{P}_H''.\tilde{P}_H = \tilde{E}_H$

Then $\tilde{P}_H' = \tilde{P}_{H.}'e$

$= \tilde{P}_H'(\tilde{P}_H.\tilde{P}_H'')$
$= (\tilde{P}_H'\tilde{P}_H)\tilde{P}_H''$ (associative law)

$= 1.\tilde{P}_H''(\tilde{P}_H.\tilde{P}_H' = e = 1)$
$\tilde{P}_H' = \tilde{P}_H''$

So, inverse of $\tilde{P}_{\mathrm{H}}$ is unique.

Hence proved theorem.

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