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A STUDY OF HEAT TRANSFER AND BROWNIAN MOTION OF NANOFLUID IN THE PRESENCE OF MAGNETIC FIELD OVER A NONLINEAR STRETCHING SHEET

Madhu Aneja[1], Sapna Sharma[2]
School of Mathematics, Thapar Institute of Engineering & Technology, India

ABSTRACT

The present analysis contains a study of steady natural convection of boundary layer flow of incompressible nanofluid over a nonlinear stretching sheet in the presence of magnetic field, Brownian motion and heat source. Koo - Klienstreuer - Li correlation is utilized to compute the effective thermal conductivity and viscosity of nanofluid. The present model includes the effect of Brownian motion on the effective thermal conductivity. Suitable similarity variables are adopted to transform the governing system of nonlinear partial differential equations to a system of coupled ordinary differential equations and solved numerically using Finite Element Method. The impact of various pertinent parameters such as Brownian motion, volume fraction, Heat source parameter and Hartmann number are analyzed on velocity and temperature. The results shows that with increase in Hartmann number velocity decreases while temperature rises. Skin friction coefficient and heat transfer coefficient are also determined for related parameters. The present study indicates that skin friction coefficient decreases with rise in thermal conductivity due to Brownian motion. In contrast to this there is insignificant rise in reduced Nusselt number.

Keywords: *Brownian motion , Convection , Magnetic field, Stretching Sheet, Finite Element Method.*

NOMENCLATURE

x, y : Cartesian coordinates
u, v : Interstital velocity component
c_p: Specific heat
G : Grashof Number
k_f : Fluid thermal conductivity
k_s : Solid particle thermal conductivity
Nu: Nusselt number
Pr: Prandtl number
T : Temperature
k_b : Boltzmann constant
k_B : Brownian motion thermal conductivity
α : Thermal diffusivity
β_f : Fluid thermal expansion coefficient
β_s : solid expansion coefficient
ϕ :solid volume fraction
υ_f: Kinematic viscosity
μ_B: Effective dynamic viscosity due to Brownian motion
θ : Dimensionless temperature
ρ : Density
eff: Effective
f :Fluid
nf :Nanofluid

INTRODUCTION

The thermal conductivity of commonly used heat transfer fluids, including oil, water and ethylene glycol mixture are inadequate to fulfill the industrial requirements of present world. Dispersing solid particles in liquids to improve its physical properties is not new and known for more than 100 years. Conventionally, the suspended particles of microns or mili meter size are considered. Due to these large size particles, the problems like abrasion and obstruction may occur. Therefore, such fluid with suspended large size particles does not have much relevance in heat transfer enhancement applications. Nanofluids have potential to reduce such problems, which was first utilized by Choi [1]. He showed that by adding small amount of nanoparticles to conventionally heat transfer liquids enhance their thermal conductivity. Due to the enhanced heat transfer performance, such fluids have various engineering applications in cooling, process industries as well as biomedical applications like cancer therapy etc. It represents the most relevant cutting edge technology being explored. One of the major advantages of using nanofluids over conventional fluids are that they are more stable, have sufficient viscosity and better wetting, spreading and dispersion properties even for low nanoparticles volume fraction [2]. The most commonly nanoparticles used in nanofluids are made up of metals like Al and Cu, Al_2O_3oxides, SiC carbides, nitrides such as AIN and SiN or non-metals like graphite, carbon and nanotubes. Generally, the base fluid is considered as water, ethylene glycol or oil. Experimentally, it has been shown that the effective heat transfer enhancement can be achieved by adding maximum 5% of the nanoparticles concentration [3]. Hence, nanofluids have diverse industrial applications in fuel cell, microelectronics, biomedicine, nuclear reactors and transportation [4].

The boundary layer flow due to stretching surface has influenced many researchers because of its use in several engineering processes, for instance, materials manufactured by extrusion, paper production and glass fibre, etc. One of the important industrial application of stretching sheet is extrusion of a polymer sheet from a die or the drawing of plastic films. During the process of making these sheets, the melt which comes out from the slit is stretched to get the desired thickness. Additionally, the rate of stretching and cooling significantly effect the quality of the final product. The linear stretching rate is not realistic, as the stretching rate has to change during the process and causing non-linear stretching. Sakiadis [5]examined the boundary layer flow of viscous fluids over a continuous solid surface moving with constant speed. After that the flow of a nanofluid over a linear stretching sheet has been analyzed by Khan and Pop [6] using Buongiorno's model [7]. Rana and Bhargava [8] also did similar kind of work for a nonlinear stretching sheet using different numerical techniques viz finite difference and finite element. The consequence of convective surface boundary condition on the boundary layer flow of nanofluid over a stretching sheet was proposed by Makinde and Aziz [9] while Mustafa et al. [10] examined the boundary layer flow for an exponentially stretching sheet via homotopy analysis method.

Magnetohydrodynamics (MHD) is the study of hydrodynamics in the presence of magnetic field. The fluid flow under the influence of magnetic field has several industrial applications such as metal casting, crystal growth, and liquid metal cooling blankets for fusion reactors, etc. Recently, the problem of MHD natural convection for nanofluids is investigated in an eccentric semi-annulus region [11]. The case of electrically conducting incompressible nanofluid is analyzed by applying the second law of thermodynamics over a porous rotating disk [12]. It is reported that magnetic rotating disk drives can enhance heat transfer in renewable energy systems. In the same direction, several other authors [13-15] studied the effect of magnetic field on flow and investigate the heat transfer properties.

Buongiorno[7] lists out all the convective properties in the comprehensive survey for nanofluids and examined seven slip mechanism to model relative velocity between the nanoparticles and base fluid. As a conclusion, two of these mechanism namely, Brownian diffusion and thermophoresis were found to be relevant for convective heat transfer. Both these mechanism were incorporated for finding the influence of nanoparticles on natural convection boundary layer flow past a vertical plate by Kuznetsov and Nield [16]. The simplest boundary conditions are incorporated in which constant temperature along with constant nanoparticle fraction are considered along the wall. The problem of natural convection past a vertical flat plate firstly proposed by Cheng and Minkowycz[17] is revisited by Nield and Kuznetsov [18] embedded in a porous medium utilizing nanofluid.It is noteworthy to mention here that nanofluid considered in this study incorporates the effects of Brownian motion and thermophoresis for the porous medium.

The purpose of the present paper is to analyze the flow and heat transfer phenomenon over a stretching sheet, with nonlinear velocity in the presence of magnetic field and heat source Q. The proposed model for the problem incorporates effect of Brownian motion on the thermal conductivity. The governing set of partial differential equations are reduced to a system of nonlinear ordinary differential equations, which is solved by Finite Element Method. The effect of various important parameters such as Hartmann number representing the effect of magnetic field, heat source parameter, volume fraction, thermal conductivity due to Brownian motion on the relevant flow variables are discussed and corresponding results are plotted by taking variation in different parameter values. Moreover, the analysis of heat transfer phenomena in terms of Nusselt number and the effect of skin friction coefficient are discussed.

MATHEMATICAL ANALYSIS

We consider a two-dimensional steady, isothermal, laminar and incompressible nanofluid passed over a surface concurring with the plane $y = 0$. The flow is assumed to be confined in $y > 0$ region. The origin is kept fixed and the surface is stretched by applying two equal forces but in opposite direction along $x -$ axis. Additionally, the size and shape of the nanoparticles are considered to follow uniform distribution. Moreover, both the fluid phase and nanoparticles are assumed to be in thermal equilibrium for the fluid stability. The fluid is electrically conducted due to an applied magnetic field B and a heat source Q is also added normal to the stretching sheet. Boussinesq approximation is used to determine the variation of density in the buoyancy term. Fig.1. represent the physical model of the problem. Under the boundary layer approximations, the basic equations governing the flow are:

Continuity Equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

Momentum Equation:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} + \frac{1}{\rho_{nf}}\left[\phi\ \rho_s\beta_s + (1-\phi)\rho_f\beta_f\right]g(T - T_\infty) - \frac{\sigma_{nf}}{\rho_{nf}}B^2 u, \quad (2)$$

Energy Equation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 u}{\partial y^2} + \frac{Q}{(\rho c_p)_{nf}}(T - T_\infty)\,. \quad (3)$$

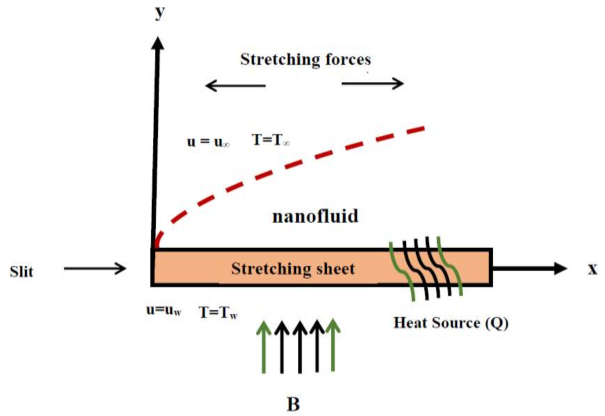


Figure 1 : Geometry of the Problem

The appropriate boundary conditions are:

$$u(x,0) = Dx^n,\ \ v(x,0) = 0,\ T(x,0) = T_w,$$
$$\text{As } y \to \infty : u \to 0,\ \ T \to \infty\,. \quad (4)$$

Here, u and v, respectively represent the velocities in the x and y - directions and T is the temperature. The properties of the nanofluidi.e, effective density $(\rho_{nf})$the effective heat capacity $(\rho c_p)_{nf}$, electrical conductivity $(\sigma)_{nf}$ and effective thermal diffusivity$(\alpha)_{nf}$ are defined as [3,19]:

$$(\rho)_{nf} = (1-\phi)\rho_f + \emptyset\,\rho_s$$

$$(\rho\, c_p)_{nf} = (1-\phi)(\rho\, c_p)_f + \emptyset\,(\rho\, c_p)_s$$

$$\frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{\left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi}{\left(\frac{\sigma_s}{\sigma_f} + 2\right) + \left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi} \quad (5)$$

$$\alpha_{nf} = \frac{k_{eff}}{(\rho\, c_p)_{nf}}$$

As discussed in the previous section, the consideration of Brownian motion significantly influences the effective thermal conductivity in the nanofluids. For simplicity, the effective thermal conductivity is considered to be consist with two components: particles's conventional static and Brownian motion [20]. Additionally, Koo and Kleinstreuer's two-component model [20] for thermal conductivity provides a flexibility to incorporate the important particle effects like size, volume fraction, temperature dependence as well as types of particle and base fluid combinations.

$$k_{eff} = k_{static} + k_B (6)$$

where, the static thermal conductivity k_{static} is computed by the Maxwell classical correlation [21] and is given by :

$$\frac{k_{static}}{\sigma_f} = 1 + \frac{\left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi}{\left(\frac{\sigma_s}{\sigma_f} + 2\right) + \left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi} (7)$$

where, ϕ is the particle volume fraction; k_f and k_s denote the thermal conductivity of the carrier fluid and particles, respectively.

The Stoke's flow is simulated around a sphere representing nano-particle to compute the enhanced thermal conductivity component affected by ambient fluid motion and produced by micro-scale convective heat transfer of a particle's Brownian motion. Now, parallel to the idea proposed by Koo [22], we use two empirical functions (β and g') to include the interaction between nanoparticles in addition to the temperature effect in the model as:

$$k_B = 5 \times 10^4\, \beta \phi \rho_f (c_p)_f \sqrt{\frac{k_B T}{\rho_p d_p}}\, g'(T,\phi), (8)$$

where, function g' captures the influence of both temperature and volume fraction. It being a continuous function of particle volume fraction is approximated as a linear function using Taylor series [23] as:

$$g'(T,\phi) = (-6.04\, \emptyset + 0.4705)T + (1722.3\, \phi - 134.63)$$

$$\beta = \begin{cases} 0.0011\, (100\, \phi)^{-0.7272} & for\ CuO \\ 0.0017\, (100\, \phi)^{-0.0841} & for\ Al_2O_3 \end{cases} (9)$$

Experimentally, the validity of the above equations is estimated in the ranges $1\,\% < \emptyset < 4\,\%,\ 300 < T < 325$ for water based nanofluids. Due to its relationship with the particle motion, the parameter β is assumed not only to depend on volume fraction but also on particle shape, temperature, and material properties of the particles/base fluid. For simplicity, in the present study we assumed the parameter β to be dependent only on volume fraction as all other dependencies mentioned here is considered in the function g'

Further, the laminar nanofluid flow in a micro heat-sinks is analyzed using the effective nanofluid thermal conductivity model [24]. Due to the micro mixing in suspensions, the effective viscosity is proposed to be the sum of static part and Brownian part as:

$$\mu_{eff} = \mu_{static} + \mu_B \qquad (10)$$

where, the viscosity of the nanofluid μ_{static} is firstly given by Brinkman [25]as$\mu_B = \frac{k_B}{k_f} \times \frac{\mu_f}{Pr}$

Now, we introduce the following similarity variables to examine the flow regime.

$$\eta = y\sqrt{\frac{D(n+1)}{2\upsilon}}\, x^{\frac{(n-1)}{2}},\ u = D\, x^n\, (\eta),\ v = -\sqrt{\frac{D\,(n+1)\upsilon}{2}}\, x^{\frac{(n-1)}{2}}\left(f + \frac{(n-1)}{(n+1)}\, \eta\, f'\right),\ \theta = \left(\frac{(T - T_\infty}{T_w - T_\infty}\right),$$

Using the above equations, the continuity Eq.(1) is satisfied automatically and the Eq.(2) and (3) with boundary conditions (4) are transformed into system of nonlinear ordinary differential equations.

$A_1A_2f^{'''} + A_3\,\theta - A_4f^{'2} + ff^{''} - A_5f^{'} = 0,$(11)

$A_6\theta^{''} + A_7\, f\,\theta^{'} + \frac{2}{(n+1)}\, Hs\,\theta = 0.$(12)

$f(0) = 0,\ \ f^{'}(0) = 1,\ \theta(0) = 1$ and $f^{'}(\infty) = 0,\ \theta(\infty) = 0,$ (13)

The coefficients $A_i\,(\,i = 1, 2, \ldots\ldots, 7)$ are dimensionless constants and provided in the Table 1:

Table 1: The coefficients A_i of the model

A_1	A_2	A_3	A_4
$\left(\frac{1}{\phi\,\frac{\rho_s}{\rho_f} + (1-\phi)}\right)$	$\left(\frac{1}{(1-\phi)^{2.5}} + \frac{k_B}{k_f} \times \frac{1}{Pr}\right)$	$\frac{2G}{(n+1)}\left(\phi\,\frac{\rho_s\beta_s}{\rho_f\beta_f} + (1-\phi)\right)$	$\frac{2n}{(n+1)}$
A_5	A_6	A_7	
$\frac{2Ha^2}{(n+1)}$	$\left(1 + \frac{3\left(\frac{k_s}{k_f} - 1\right)\phi}{\left(\frac{k_s}{k_f} + 2\right) - \left(\frac{k_s}{k_f} - 1\right)} + \frac{k_B}{k_f}\right)$	$\Pr\left(\phi\,\frac{(\rho c_p)_s}{(\rho c_p)_f} + (1-\phi)\right)$	

The prime represents the derivatives with respect to η .The parameter $G = \frac{g\,K\beta_f\,(T_w - T_\infty)}{D^2 x^{(2n-1)}}$ is modified Grashof number also called convection parameter because it occurs due to convection, $\Pr = \frac{(\rho c_p)_f \nu_f}{k_f}$ is the Prandtl number, $Hs = \frac{Q_v}{k_f}\frac{1}{Dx^{n-1}}$ is the modified Heat source parameter and $Ha = \frac{B}{D\,x^{n-1}}\sqrt{\frac{\sigma_{nf}}{\rho_{nf}}}$ is the modified Hartmann number. Further, the shear stress at the surface of the sheet can be computed as

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right) = -\frac{D\,\mu_f x^{(3n-1)/2} f^{''}(0)}{(1-\phi)^{2.5}}\sqrt{\frac{D(n+1)}{2\,\nu_f}} \quad (14)$$

The parameter of interest is the skin friction coefficient C_f, given by

$$C_f = \frac{\tau_w}{\rho\, u^2{}_w} \quad (15)$$

Putting the value of τ_w in C_f, we obtain

$$C_f\,(Re_x)^{\frac{1}{2}}\sqrt{\frac{n+1}{2}}\frac{f^{''}(0)}{(1-\phi)^{2.5}} \quad (16)$$

Where $Re_x = \frac{u_w\,x}{\nu_f}$

The heat flux at the surface is calculated as

$$q_w = -k_{nf}\frac{\partial T}{\partial y} = -k_{nf}\,(T_w - T_\infty)\sqrt{\frac{D(n+1)}{2\,\nu_f}}\,x^{\frac{n-1}{2}}\theta^{'}(0), \quad (17)$$

And Nusselt Number Nu_x is given by

$$Nu_x = \frac{q_w x}{k_f\,(T_w - T_\infty)}. \quad (18)$$

Putting the value of q_w in Nu_x , we obtain

$\frac{Nu_x}{\sqrt{Re_x}} = -\frac{k_{nf}}{k_f}\sqrt{\frac{(n+1)}{2}}\,\theta^{'}(0)$(19)

Table 2: Thermophysical properties of base fluid and different nanoparticles at room temperature

Physical Properties	**Fluid phase (water)**	**CuO**	**Al_2O_3**
C_p (J/Kg K)	4179	531.8	765
ρ (kg/m^3)	997.1	6320	3970
k (W/mK)	0.613	76.5	40.0
$\beta \times 10^{-5}$ (1/K)	21.0	1.8	0.85
d_p(nm)	-	29	47

SOLUTION OF THE PROBLEM

The governing equation of the flow problem along with the boundary conditions are coupled non-linear ordinary differential equations. These are solved by efficient numerical technique i.e., Finite Element Method.

RESULTS & DISCUSSION

Finite Element method is used to solve the governing nonlinear system of Eq. (11) and (12) with boundary condition (13). Various important and interesting parameters such as volume fraction ϕ, non-linear parameter n, modified Grashof number G, Prandtl number Pr, Hartmann number Ha, Heat source parameter Hs, Brownian motion thermal conductivity k_B are discussed to see their influence on dimensionless velocity $f^{'}(\eta)$ and dimensionless temperature $\theta(\eta)$. The volume fraction ϕ of nanoparticles ranges $0\% < \phi < 4\%$ in this study. The thermophysical properties of the fluid and solid are demonstrated in table 1. Results for velocity and temperature profiles are plotted by taking different values of parameters of the problem. We have taken Al_2O_3as working fluid in our study. For the validation of code, we have made comparison with the results for the reduced Nusselt number obtained by Wang [26], Gorla and Sidawi[27] for different values of Pr and is presented in Table 3. Fig (2a-2b) interpret the effect of volume fraction ϕ on the velocity and temperature field. The numerical results of the present study indicate that velocity and temperature of nanofluid increases remarkably with the increasing volume fraction of nanoparticles. From physics point of view, it can be explained that the random movement of the particle increases with rise in nanoparticles volume fraction, as a result the energy exchange rates within the fluid and consequently enhances the thermal dispersion in the flow of nanofluids.

The impact of Hartmann number Ha on the velocity and temperature profiles is shown in Fig (3a - 3b). Fig 3a depicts that velocity declines within the boundary layer with increase in Hartmann number. This behaviour is observed due to the fact that a retarding body force known as Lorentz force is introduce with increase in magnetic field, which act in the transverse direction to the applied magnetic field. Hence momentum boundary layer becomes thinner as Ha rises. Incontrast to this behaviour, it is displayed in Fig (3b) that temperature enhances insignificantly, as the nature of Lorentz force is resistive, thus it opposes the fluid motion. Due to this heat is produced which makes the thermal boundary layer thick.

The influence of heat source parameter on velocity and temperature profile is described in Fig (4a - 4b). It is examined that when heat source parameter numerically increases, temperature decreases within the boundary layer as shown in Fig (4b). Unlike to temperature profile, from Fig 4(a) it revel that velocity overshoots in the vicinity of boundary layer and decreases as Hs goes on increasing numerically.

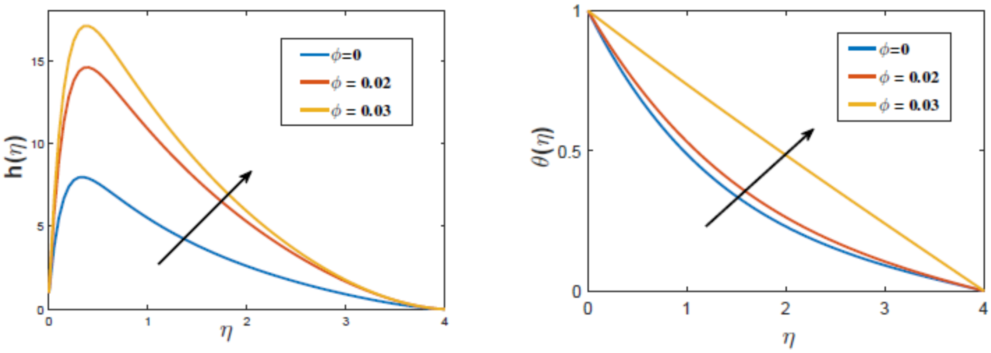
The thermal conductivity due to Brownian motion is k_B calculated for different values of temperature T. Fig (5a - 5b) demonstrates the effect of k_B on velocity and temperature field. Fig (5a) shows the effect of thermal conductivity due to Brownian motion on velocity. From this Fig, it is noticed that velocity increases in the vicinity of the boundary layer upto some extent and after that it decreases as k_B serves to warm the boundary layer and simultaneously exacerbates particle deposition away from the fluid regime or onto the surface. It is also observed that Brownian motion thermal conductivity k_B does exert a significant enhancing influence on temperature profile. Fig (5b) illustrates that temperature profile rises as k_B increases.

The effect of various pertinent parameters on skin friction coefficient and reduced Nusselt number are described in Table (4 -7). The variation of skin friction coefficient and Nusselt number for varying nonlinear stretching parameter , Hartmann number and Prandtl number is shown in Table 4. As Hartmann number increases skin friction coefficient increases for fixed value of nonlinear stretching parameter, same happens with the Nusselt number. But with increasing nonlinear stretching parameter skin friction coefficient decreases with increase of Hartmann number, reverse in the case of reduced Nusselt number. Table 5 depicts the behaviour of reduced Nusselt number with varying Heat source parameter, nonlinear stretching parameter and Prandtl number while keeping other parameter fixed. It is observed that rate of heat transfer increases with decrease in Hs and increase in Pr while it decreases with increase in nonlinear stretching parameter. The variation of skin friction coefficient with respect to volume fraction is shown in Table 6. As volume fraction increases velocity increases within the boundary layer due to this skin friction coefficient decreases. Nusselt number and skin friction coefficient is also calculated for different values of Brownian motion thermal conductivity and is shown in Table 7. Skin friction coefficient increases with increase in k_B while reverse effect is observed on reduced Nusselt number.

FIGURES AND TABLES

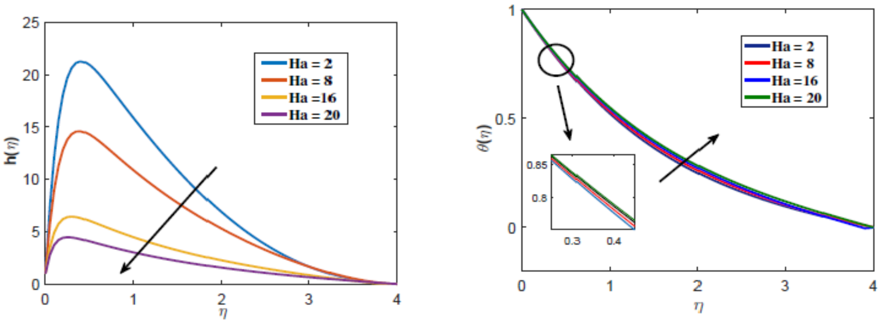
Table 3: Comparison of results for reduced Nusselt number for Pr with $n = 1$, $G = 0$, $Hs = 0$, $Ha = 0, k_B = 0$, $\phi = 0$

Pr	Present Result $\theta'(0)$	Wang [26] $\theta'(0)$	Gorla and Sidawi [27] $\theta'(0)$
0.07	0.0657	0.0656	0.0656
0.7	0.1694	0.1691	0.1691
2.0	0.9108	0.9114	0.9114
7.0	1.8918	1.8954	1.8954



(a) Effect of volume fraction ϕ on velocity (b) Effect of volume fraction ϕ on temperature

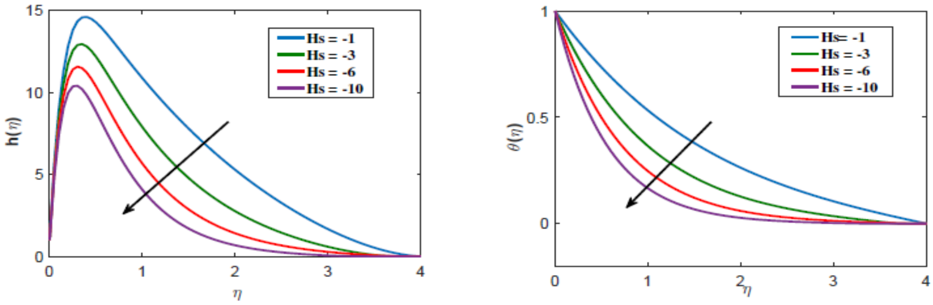
Figure 2: Effect of volume fraction ϕ on velocity and temperature when $T = 315$, $G = 1000$, $Pr = 0.7$, $Hs = -1$, $Ha = 8$



(a) Effect of Hartmann number on velocity

(b) Effect of Hartmann number on temperature

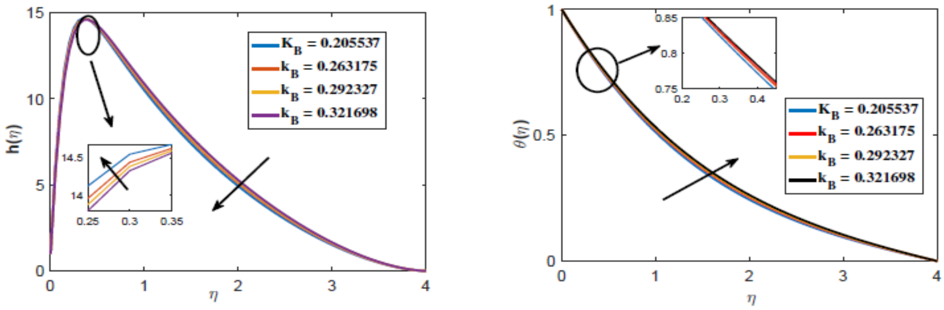
Figure 3: Effect of Hartmann number on velocity and temperature when $\phi = 0.02$, $n = 3$, $T = 315$, $G = 1000$, $Pr = 0.7$, $Hs = -1$



(a) Effect of Heat source parameter on velocity

(b) Effect of Heat source parameter on temperature

Figure 4: Effect of heat source on velocity and temperature when $\phi = 0.02$, $n = 3$, $G = 1000$, $Pr = 0.7$, $Ha = 8$, $T = 315$



(a) Effect of k_B on velocity

(b) Effect of k_B on temperature

Figure 5: Effect of thermal conductivity due to Brownian motion k_B on velocity and temperature when $\phi = 0.02$, $G = 1000$, $n = 3$, $Pr = 0.7$, $Hs = -1$, $Ha = 8$

Table 4: *Results for skin friction coefficient and coefficient of Nusselt number for* $\phi = 0.02$ *with* $G = 0,\ T = 315,\ Hs = -1.0$

		$f''(0)$				$\theta'(0)$			
Pr	n	$Ha=0$	$Ha=8$	$Ha=16$	$Ha=20$	$Ha=0$	$Ha=8$	$Ha=16$	$Ha=20$
0.7	0	0.481242	5.93198	8.73978	9.58074	1.06476	1.06389	1.06375	1.06372
	3	0.839033	3.59869	5.94284	6.83336	0.548311	0.547246	0.546993	0.546925
	5	0.869714	3.07562	5.18126	6.02361	0.45091	0.449803	0.449492	0.44942
	10	0.896673	2.43178	4.159	4.89824	0.338306	0.337173	0.336766	0.33667
7.0	0	0.607556	6.87675	9.62261	10.3637	1.07446	1.06585	1.06468	1.06441
	3	1.04735	4.32687	6.88753	7.79957	0.561566	0.551381	0.549165	0.548651
	5	1.08492	3.72609	6.07994	6.97166	0.46543	0.454867	0.452207	0.451587
	10	1.11774	2.97283	4.95846	5.77359	0.354704	0.343972	0.340512	0.339702
70.0	0	0.62651	7.00356	9.73166	10.4572	1.17472	1.0855	1.07131	1.36362
	3	1.07815	4.42962	7.01431	7.9263	0.704561	0.594107	0.571238	0.566056
	5	1.1167	3.81878	6.20269	7.09864	0.624659	0.508185	0.480095	0.473746
	10	1.15037	3.05078	5.07004	5.8939	0.53876	0.41805	0.379832	0.371133

Table 5: *Results for coefficient of Nusselt number for* Hs, Pr *and* n, $G = 0,\ \phi = 0.02,\quad Ha = 8.$

		$\theta'(0)$			
Pr	n	$Hs=-1$	$Hs=-3$	$Hs=-5$	$Hs=-10$
0.7	0	1.06389	1.77173	2.22766	3.00468
	3	0.547246	0.928327	1.18204	1.6305
	5	0.337173	0.571064	0.730689	1.01732
	10	1.03581	1.10871	1.21406	1.34781
7.0	0	1.06585	1.77342	2.2292	3.006
	3	0.551381	0.931978	1.18541	1.63346
	5	0.454867	0.769271	0.980245	1.35588
	10	0.343972	0.577105	0.736303	1.02228
70.0	0	1.0855	1.79035	2.24461	3.01911
	3	0.594107	0.969012	1.21947	1.66313
	5	0.508185	0.815272	1.02252	1.39276
	10	0.41805	0.640416	0.794303	1.07285

Table 6: *Results for skin friction coefficient and reduced Nusselt number for* ϕ *with* $n = 3$, $G = 1000,\ Pr = 0.7,\ Ha = 8,\ T = 315.$

ϕ	$f''(0)$	$\theta'(0)$
0	-45.8812	0.682827
0.02	-80.9752	0.577319
0.03	-96.5758	0.577892

Table 7: *Results for skin friction coefficient and reduced Nusselt number for* k_B *with* $n = 3$, $G = 1000$, $Pr = 0.7$, $Hs = -1$, $\phi = 0.02$, $Ha = 8$

k_B	$f''(0)$	$\theta'(0)$
0.205537	-77.1643	1.07821
0.263175	-76.002	1.07407
0.292327	-75.3716	1.07214
0.321698	-74.7132	1.07032

CONCULSION

The influence of heat source Q and thermal conductivity due to Brownian motion on steady MHD fluid flow past a nonlinear stretching sheet is investigated in this paper. Effects of volume fraction ϕ, Hartmann number Ha, nonlinear stretching parameter n, Prandtl number Pr, Heat source parameter Hs, Brownian motion thermal conductivity k_B on velocity profiles, temperature profiles, skin friction coefficient and heat transfer characterstics at the wall are studied. The main observation of the present study are as follows:

- The dimensionless velocity rises with increase in concentration of nanoparticles Ha. It is also observed that when Heat source parameter increases in magnitude, velocity overshoots in the vicinity of boundary layer and after that it declines, while it decreases as the Hartmann number Ha escalate. The behaviour of velocity profile is somewhat different for Brownian motion thermal conductivity. Firstly velocity enhances with increase in k_B upto some extent and then it decreases.
- The dimensionless temperature increases with increase in concentration of nanoparticles, Hartmann number Ha and Brownian motion thermal conductivity k_B. Temperature profile declines when Heat source parameter Hs increases numerically.
- The skin friction coefficient decreases with increase in nanoparticles concentration, nonlinear stretching parameter n while reverse effect is observed with variation in Hartmann number Ha and Brownian motion thermal conductivity k_B.
- The heat transfer rate decreases at the surface with increase in Hartmann number Ha and Prandtl number Pr while reverse behaviour is observed with increase in nonlinear stretching parameter n. It is also observed that rate of heat transfer rises when Heat source parameter Hs increases in magnitude.

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